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AN INVESTIGATION OF WEB STRESSES IN REINFORCED CONCRETE BEAMS

PART II

RESTRAINED BEAMS

BY

FRANK E. RICHART

AND

LOUIS J. LARSON

BULLETIN NO. 175

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BY

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ENGINEERING EXPERIMENT STATION
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PART II

I. Introduction

1. *Purpose and Scope of Tests.*—The tests reported in this bulletin form a part of an extended investigation of web stresses in reinforced concrete beams. One part of the investigation, which was confined to a study of the action of simple beams, has already been described in Bulletin 166, Engineering Experiment Station, University of Illinois. The part with which the present bulletin is concerned was planned as a study of the action of web reinforcement in overhanging or restrained beams in which there are both positive and negative bending moments. Since the two parts of the investigation are similar in many respects, frequent reference will be made to Bulletin 166.

A large proportion of the reinforced concrete beams used in buildings and other structures are continuous or at least partially restrained beams. The simple beam has been used for test purposes principally because of ease of making and testing; however, there has been some question as to whether the data obtained from tests of simple beams will apply equally well to restrained beams. Part of the uncertainty is due to the general use of the shearing unit stress as a measure of the diagonal tension in the web. The diagonal tension is actually a function of both shearing stress and flexural stress. Although a simple beam and a restrained beam may have identical vertical shear diagrams under a given loading, it is obvious that the bending moments and the horizontal fiber stresses will be greatly different in the two cases; hence the diagonal tension and the location and effect of diagonal cracks may be expected to vary with the different conditions of restraint.

It is apparent that tests of restrained beams should serve several purposes: they should give direct information as to whether the equations derived for web stresses in simple beams apply equally well to continuous beams; they should indicate differences in behavior and in manner of failure; and they should give valuable data regarding the effectiveness of the arrangement of web reinforcement in the vicinity of supports and load points. Very few tests have been made of continuous beams with the specific object of studying web resistance. Tests made by Withey* on overhanging beams gave some quantitative information on the action of different kinds of web reinforcement. Other tests by Scheit and Probst† and by Bach and Graf‡ on continuous beams and frames furnish a small amount of incidental information as to web stresses.§

The continuous or restrained beam in general is subjected to higher shearing stresses in proportion to the flexural stresses than a simple beam. Thus, if the beam section is governed by consideration of flexural stresses, the shearing unit stress in the restrained beam will be considerably higher than in the simple beam. It is important to know how high a shearing stress may be allowed in design and under what conditions of reinforcement satisfactory action of the beam may be assured. At present, rather high allowable shearing stresses are being advocated when certain requirements are met as to the design of the longitudinal and web reinforcement. The results of the following tests should serve to indicate some of the requirements necessary to the attainment of high shearing stresses.

The tests described in this bulletin were made on fifty-nine large rectangular beams, 18 feet long, 8 inches wide, and 15 inches in effective depth. Extensive strain measurements were made at regular increments of load on both longitudinal and web reinforcement. One group of 17 beams made in 1911 had web reinforcement consisting mainly of stirrups, with a few bent-up bars; the other group of 42 beams made in 1917 had web reinforcement consisting mainly of bent-up bars. These beams were tested on a 12-foot span, with ends overhanging, and while they are described as restrained beams it may be noted that the reactions and moments were statically determinate. Thus, while the moment at the support was twice as great as at mid-span, corresponding to the moment distribution obtaining in a beam having fixed ends and a constant moment of inertia of section, still there was no opportunity for the bending moment at any point to be relieved by a local yielding of the material, as would be the case in a continuous beam. This has the advantage, however, of maintaining constant relations between flexural and shearing stresses, so that in so far as web stresses are concerned the method of loading used seems very satisfactory and will give results comparable with those to be found in continuous structures.

*Withey, M. O., "Tests of Plain and Reinforced Concrete, Series of 1906," Bulletin of Engineering Series, Vol. 4, No. 1, Univ. of Wis., 1907.

†Scheit, H. and Probst, E., "Untersuchungen an durchlaufenden Eisenbetonkonstructionen," Berlin, 1912.

‡Bach, C. and Graf, O., "Versuche mit Eingespannten Eisenbetonbalken," Heft 45, Deutscher Ausschuss für Eisenbeton, Berlin, 1920.

§For bibliography of tests of web stresses in simple beams, see Univ. of Ill. Eng. Exp. Sta. Bul. 166, 1927.

2. *Acknowledgments.*—The investigation was carried on under the direction of PROF. ARTHUR N. TALBOT, who was responsible for the broad and comprehensive scope of the tests. He initiated the work, planned the tests in most details, directed the conduct of the tests and gave much time and thought to the study and interpretation of the results. Especial acknowledgment is made to Professor Talbot for helpful criticism and assistance in the preparation of this bulletin.

D. A. ABRAMS, H. F. GONNERMAN, and W. A. SLATER, former members of the Engineering Experiment Station Staff, gave assistance in the planning of tests and in supervising the preparation and testing of all beams. W. I. HARGIS, Graduate Student in Theoretical and Applied Mechanics in 1910-11, and L. J. LARSON, Research Fellow in the Engineering Experiment Station in 1915-1917, used parts of the investigation as theses for their master's degrees.

The tests were conducted as a part of the work of the Engineering Experiment Station of the University of Illinois of which DEAN M. S. KETCHUM is the director, and of the Department of Theoretical and Applied Mechanics, of which PROF. M. L. ENGER is the head.

3. *Analytical Relations.*—A general summary or résumé of the analytical relations and of equations in current use in the literature of the subject of web stresses was given in Bulletin 166; it will be sufficient to repeat here only the working equations to which reference may frequently be made in the discussion of the results of tests.

An exact analysis of web stresses in reinforced concrete is admittedly impracticable; instead, approximate relations and analogies have been used to furnish a fairly definite conception of the action of the web reinforcement and to give a basis for rules and methods of design. It is well to keep in mind the approximate nature of the equations that are in use and to note the empirical character of such additional relations as have been determined from the results of tests.

The following notations will be used throughout the bulletin:

a = spacing of web reinforcing bars, measured normal to their direction.

a_v = cross-sectional area of web reinforcement, in one plane or layer.

b = width of beam.

d = effective depth of beam.

f_v = tensile unit stress in web reinforcement.

jd = lever arm of resisting couple or resisting moment.

$K = \frac{v}{rf_v}$ = efficiency factor for web reinforcement.

m = number of bars used in computing bond stress.
o = periphery of one longitudinal bar.
r = ratio of web reinforcement $= \frac{a_v}{ab}$
s = spacing of web reinforcement, measured along axis of beam.
S = portion of length of web of beam effectively reinforced by bent-up bars.
u = nominal bond stress.
v = nominal shearing unit stress in concrete.
V = total external vertical shear on beam.
α = angle of inclination of web reinforcing members to horizontal.
θ = angle of inclination of diagonal compression in web to horizontal.

It is common practice to use the shearing unit stress in reinforced concrete beams as a measure of the diagonal tension. According to the usual assumption that no longitudinal tension exists in the concrete, and that compressive stresses vary directly as the distance from the neutral axis, the resulting shearing unit stress is zero at the compression surface, increases according to a parabolic curve to a maximum at the neutral axis, and is constant from the neutral axis to the plane of the tension reinforcement. The maximum value of this nominal shearing unit stress is given by the expression

$$v = \frac{V}{bjd} \tag{1}$$

Corresponding to equation (1) a similar equation gives the value of the nominal bond stress between longitudinal bars and concrete

$$u = \frac{V}{mojd} \tag{2}$$

From equations (1) and (2) it is seen that $vb = mou$, or that the shearing stress on a horizontal section of the beam of width b and length Δs is equal to the total bond stress or change in longitudinal tensile stress in the same distance Δs. It follows that any action which disturbs the bond stress u, such as a slipping between concrete and steel, produces a similar variation in v. The term nominal stress is used to indicate that the actual stress may vary considerably from that given by the equation.

An analysis of stress in tension web members has frequently been made by showing the analogy between such web members and web members of a truss, in which the top chord is formed by the compression zone of the concrete, the bottom chord by the longitudinal

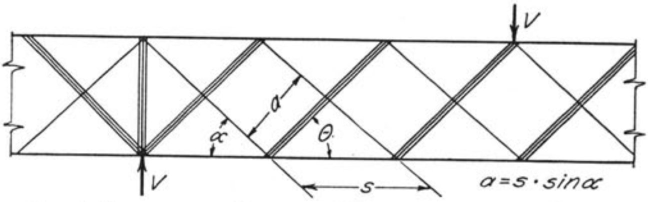


FIG. 1. TRUSS WITH INCLINED TENSION AND COMPRESSION WEB MEMBERS

reinforcement, and the diagonal compression web members by portions of the concrete web of the beam. Figure 1 shows such a truss, in which the tension web members make an angle α and the compression web members an angle θ with the horizontal. The web members are connected to the top and bottom chords. For such a truss the unit stress in any tension web member may be shown to be

$$f_v = \frac{Va}{Ka_v\,jd} \tag{3}$$

where $$K = (\sin\alpha\cot\theta + \cos\alpha)\sin\alpha \tag{4}$$

Equation (3) may be expressed in different notation by substituting $r = \frac{a_v}{ab}$ and $bv = \frac{V}{jd}$, whence

$$f_v = \frac{v}{rK} \tag{5}$$

It is usually assumed, and the assumption is substantiated by the location and direction of cracks in test beams, that the angle θ may be taken at a constant value of 45 deg., so that

$$K = (\sin\alpha + \cos\alpha)\sin\alpha \tag{6}$$

For stirrups inclined at 45 deg. or for vertical stirrups ($\alpha = 45$ or 90 deg.) it is seen from equation (6) that K becomes equal to unity, so that

$$f_v = \frac{Va}{a_v\,jd} = \frac{v}{r} \tag{7}$$

In some cases it is also desirable to know the magnitude of the diagonal compressive stress. Referring to Fig. 1, the vertical component of stress in the tension web member must equal the vertical component of the stress in the diagonal compression member. For values of θ equal to 45 deg. and for tension web members inclined at

45 deg. the magnitude of the total diagonal compression over a length s is $\frac{0.71\,Vs}{jd}$.

For the same value of θ and with vertical web members the total diagonal compression is $\frac{1.41\,Vs}{jd}$, or twice as much as when the web members are inclined at 45 deg. to the horizontal. Diagonal compression stresses are not usually important unless thin webs or high amounts of web reinforcement are used.

The analogy between a reinforced concrete beam having bent-up bars and the truss of Fig. 1 is not so apparent as for the beam with stirrup web members. However, if the bars are bent up at regular intervals, and fully reinforce the region subject to vertical shear, the stresses in the bent-up bars may be expected to follow the same laws as do the stresses in stirrups. The use of bent-up bars in restrained beams presents a number of unusual conditions. The variation in bond stresses in the bent-up bars and in the remaining straight bars becomes quite complex. As will be seen from the tests to follow, bond stresses become very important in this type of beam.

In equations (3) to (7) it has been tacitly assumed that the stress produced in the web steel was due to the entire vertical shear. It is well known that measured stresses in the web steel are quite generally less than those calculated by use of the full shear, and two general assumptions are frequently made: (1) that the unbroken portion of the concrete carries a certain fixed *amount* of the shear, and (2) that the concrete carries a certain *proportion* of the shear. Both assumptions are confirmed, to some extent, by test data; the conclusions in Bulletin 166 were that the load-stress curve for stirrups and bent-up bars agreed fairly well with an equation based on an assumption of the first type, such as

$$v = C + rf_v \tag{8}$$

However, the value of C varied considerably for different series of tests. For stresses at or near the maximum load on the beam, an equation of the second type in which the stresses in web members correspond to a certain proportion of the shear (depending upon the percentage of web reinforcement) was found to be applicable. The equation is

$$v = (0.005 + r)\,f_v \tag{9}$$

This equation was found to agree fairly well at maximum loads for several series of tests. The equation was first derived by Slater, Lord, and Zipprodt,* and was given as the relation best expressing the results of a large series of tests of simple beams with varying amounts of web reinforcement.

The applicability of these formulas to the results of the tests included in this bulletin will be treated in the discussion of the test data.

II. Materials, Test Beams, and Tests

4. *Materials.*—The materials used in the test beams were similar in character and quality to those used in the tests of Bulletin 166, as were also the general methods of making and testing the beams. Information as to the physical properties of the concrete materials and of the concrete control specimens made with the test beams is given in Table 1.

Table 1

Data of Concrete and Concrete Materials

Materials: Portland cement, Universal (Series of 1911 and 1917) and Lehigh (Series of 1911).
Torpedo sand from Attica, Indiana, 0 to ¼-in. size.
Limestone from Kankakee, Illinois, ¼ to 1-in. size, Series of 1911.
Gravel from Attica, Indiana, ¼ to 1-in. size, Series of 1917.

Item			Series of 1911	Series of 1917
Beam No.			371.1–378.2	380.1–400.2
Tensile Strength of Briquets, lb. per sq. in.	Neat Cement	Age, 7 days	719	613
		Age, 28 days	805	769
	Ottawa Sand Mortar 1:3, by weight	Age, 7 days	248	232
		Age, 28 days	329	318
	Attica Sand Mortar 1:3, by weight	Age, 7 days	....	345
		Age, 28 days	....	465
Average Proportions of Mixture		By volume	1:2:4	1:2:4
		By weight	1:2.1:3.6	1:2.2:4.0
Per cent of Mixing Water, by weight			9.2	8.0
Water-cement ratio, by volume			0.92	0.88
Control Specimens	Comp. Strength, lb. per sq. in.	6-in. cubes	2250	....
		6-in. by 12-in. cylinders	....	3140
	Age, days		65	61
Control Beams, 6-in. by 8-in. by 40-in.	Modulus of Rupture, lb. per sq. in.		320	....
	Age, days		65	....

*Slater, W. A., Lord, A. R., and Zipprodt, R. R., "Shear Tests of Reinforced Concrete Beams," Technologic Paper No. 314, U. S. Bureau of Standards, 1926.

TABLE 2
SIEVE ANALYSES OF FINE AND COARSE AGGREGATES
(Made with Tyler Standard Screen Scale Sieves)

Kind of Aggregate	Series	Per cent by Weight Passing Given Sieve								
		100	48	28	14	8	4	⅜-in.	¾-in.	1-in.
Sand.........	1911	2	8	28	62	85	99	100	...	...
Sand.........	1917	3	7	18	42	71	95	100	...	...
Limestone....	1911	...	...	...	1	2	4	35	98	100
Gravel.......	1917	...	...	...	...	...	1	14	73	100
Width of Sieve Opening—in		0.0058	0.0116	0.0232	0.046	0.093	0.185	0.371	0.742	1.05

In the series of 1911 two brands of Portland cement were used, Lehigh and Universal; in the 1917 tests only Universal was used. The Lehigh cement was purchased from a local dealer, while the Universal cement was furnished to the University by the manufacturers for experimental purposes. Tests of these cements as to fineness, time of set, and tensile strength showed that they complied with standard specifications. The results of briquet tests of the cements are given in Table 1.

The fine aggregate used was a clean, coarse torpedo sand from Attica, Indiana. The coarse aggregate used in 1911 was a broken limestone from Kankakee, Illinois; that used in 1917 was a washed gravel from Attica, Indiana. Both stone and gravel ranged from ¼ in. to 1 in. in size. Sieve analyses of the aggregates are given in Table 2.

The concrete used in the test beams and control specimens was mixed in the proportions 1:2:4 by loose volume. All concrete was machine mixed; usually each batch was mixed four minutes. The work of mixing concrete and of pouring specimens was done by experienced workmen under the supervision of a member of the Experiment Station Staff. Data regarding the properties of the concrete are given in Table 1.

Mild steel was used as reinforcement in all but six of the fifty-nine beams included in this bulletin. In these six beams fabricated reinforcing units were used. The Corrugated Bar Company of St. Louis, Mo., furnished two units made up of high carbon corrugated round longitudinal steel and corrugated square stirrups rigidly attached, and two units having the same longitudinal bars, but having rigidly attached stirrups of plain round mild steel. These units were

Table 3
Tension Tests of Reinforcing Steel

Series	Beam No.	No. of Specimens Tested	Description of Bar	Unit Stress at Yield Point, lb. per sq. in.	Ultimate Tensile Strength, lb. per sq. in.	Per cent Elongation in 8 inches
1911	371.1	3	¾-in. round	41 900	61 100	27.8
	372.1	2	¾-in. round	33 200	......	....
	373.1	3	¾-in. round	34 400	......	....
	373.2	3	¾-in. round	34 200	......	....
	374.1	2	¾-in. round	35 100	......	....
	375.1	2	⅝-in. round*	60 500	......	....
	375.3	2	½-in. round*	51 200	90 600	15.3
	375.3	4	⅝-in. round*	64 200	101 800	14.3
	375.3	3	¾-in. round*	67 900	107 600	9.0
	376.2	3	¾-in. corr. round*	56 600	......	....
	376.6	3	¾-in. corr. round*	54 800	......	....
	377.1	3	¾-in. round	40 100	62 700	28.8
	377.2	3	¾-in. round	42 400	62 800	29.7
	378.1	3	¾-in. round	40 700	61 000	25.8
	378.2	3	¾-in. round	42 400	64 300	25.0
1911	371.1	4	¼-in. round	35 400	51 800	....
	372.2	4	¼-in. round	38 600	......	....
	373.2	5	¼-in. round	37 800	......	....
	375.3	5	¼-in. round	44 800	60 800	....
	376.2	4	0.20-in. round	41 300	......	....
	376.6	4	¼-in. corr. square*	60 200	......	....
	377.1	4	¼-in. round	38 700	52 600	....
	377.2	5	¼-in. round	43 300	56 200	....
	378.1	5	¼-in. round	39 400	53 200	....
	378.2	6	¼-in. round	36 400	49 500	....
1917	380.2	5	⅝-in. round	37 900	59 100	29.3
	381.2	8	⅝-in. round	36 100	57 600	29.7
	382.2	5	⅝-in. round	38 500	57 200	32.4
	387.2	4	⅝-in. round	39 500	58 100	30.2
	388.2	5	⅝-in. round	35 700	57 500	29.4
	389.2	6	⅝-in. round	37 700	57 500	30.1
	390.2	3	⅝-in. round	38 400	58 000	30.0
	393.1	4	⅝-in. round	40 600	58 500	29.6
	394.1	4	⅝-in. round	39 100	56 600	30.2
	397.2	8	⅝-in. round	36 100	57 600	29.7
	398.1	4	⅝-in. round	37 700	58 300	29.1
	398.2	2	⅝-in. round	38 200	56 800	28.4
	399.1	2	⅝-in. round	36 900	60 600	26.2
1917	All	15	⅜-in. round	45 100	63 000	26.8

*High carbon steel; all others mild steel.

used in beams 376.5 and 376.6 and in 376.1 and 376.2, respectively. The American System of Reinforcing of Chicago, Illinois, furnished two units consisting of plain round high carbon longitudinal bars and plain round mild steel stirrups, which were used in beams 375.1 and 375.3. The mild steel bars used in the series of 1917 were furnished by the Illinois Steel Company, Chicago, Illinois. Details of the make-up and arrangement of the various types of reinforcement are given in the various tables and diagrams to follow. Average physical properties of the reinforcing steel are given in Table 3.

5. *Test Beams.*—The test beams were all of the overhanging type, 18 feet long, 8 inches wide, and 15 inches in effective depth; they were

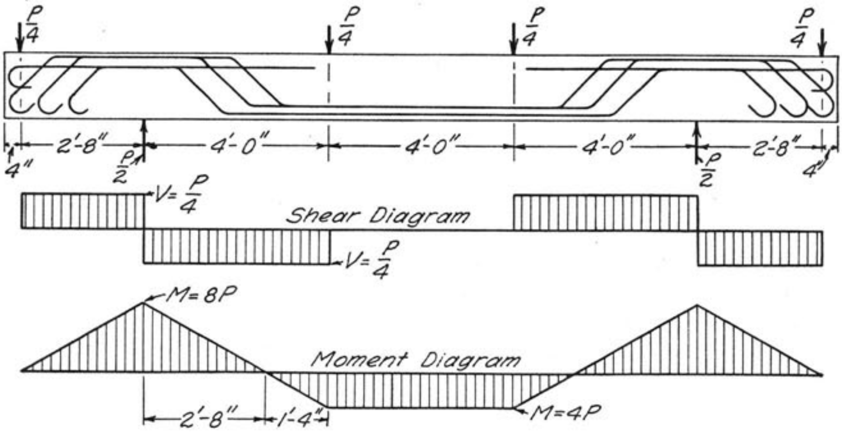


FIG. 2. SKETCH OF TYPICAL BEAM, SHEAR, AND MOMENT DIAGRAMS

tested on a 12-foot span with a 3-foot overhang at each end. Four equal loads were applied; two at the one-third points of the span, and two on the projecting ends, 2 ft. 8 in. outside the supports. This arrangement of loads produced a negative moment at the support twice as great as the positive moment at mid-span, a condition that should obtain in a beam of constant section having fixed ends and subjected to equal loads at the one-third points of the span, or subjected to a uniform load. Figure 2 shows a sketch of a typical test beam, with the accompanying moment and shear diagrams produced by the given loading.

The longitudinal reinforcement in the beams of the series of 1911 varied from 0.7 to 1.4 per cent at both support and mid-span, the bars were generally ¾-in. round and were placed in one layer. The web reinforcement consisted of stirrups and bent-up bars in various arrangements. In the series of 1917 the longitudinal reinforcement was about 2.0 per cent at the supports and from 1.0 to 2.0 per cent at mid-span. The bars were all ⅝-in. round, and were generally placed in two layers. A large number of bars were thus available for use as web reinforcement, which consisted almost entirely of bent-up bars. A few vertical stirrups were used in the series. Details of all of the reinforcement are given in tabular form in Sections 8 and 15.

Anchorage was provided principally by the use of hooks on both longitudinal bars and on stirrups. In four beams of the 1911 series, the ends of bars which projected beyond the ends of the beams were threaded and anchored by means of steel bearing plates and nuts, tightened immediately before the tests. In the series of 1917 especial attention was paid to the design of hooks and bends, and all were made with a 3-inch radius.

6. *Making and Testing of Beams.*—All beams were poured in wood forms resting on a strip of building paper on the concrete floor of the laboratory. The reinforcing steel was carefully wired in position, corks were nailed to the forms in proper places to expose the reinforcement for gage lines, and supporting bars were placed to hold all reinforcement at the desired level. In the earlier series these rigid supporting bars were left in the beam, with the result that the concrete showed a tendency to settle away from beneath the bars; in the 1917 tests the supporting bars were removed soon after the beam was poured. The concrete was well tamped and spaded around the reinforcing bars.

The forms were removed four to seven days after the beams were poured and the beams were covered with wet burlap and sprinkled daily. The beams were tested at an age of approximately 60 days, except six beams made in 1911, which were tested after about two years.

In preparation for testing all beams were whitewashed and strain gage holes were prepared. The 1911 tests were made in a Riehle testing machine of 600 000-lb. capacity, the 1917 tests in an Olsen machine of 300 000-lb. capacity. Views of beams during tests are shown in Figs. 3 and 4. In both cases load was applied through a series of I-beams and rollers and transmitted to the specimen through 4-in. by 8-in. bearing plates set in plaster of Paris. Load was applied at the rate of about 0.06 in. per min.

Various forms of the Berry strain gage were used in measuring strains in the reinforcement, a complete series of strain measurements being taken at each increment of load. There were usually 50 to 75 gage lines on each beam, and measurements were taken at from four to nine increments of load. The strain readings were taken on 4-in. and 6-in. gage lines with a precision of 0.00001 to 0.00002 in. per in. In the series of 1911 the slipping of reinforcing bars was also observed, measurements being taken by means of Ames micrometer gages graduated to 0.0001 in.

Auxiliary test pieces made of the same batches of concrete as were used in the test beams were stored with the beams and tested at corresponding ages. In the series of 1911 the auxiliary control specimens were 6-in. cubes and 6-in. by 8-in. by 40-in. plain concrete beams. In the series of 1917 the standard 6-in. by 12-in. test cylinders were used. Data of these auxiliary specimens are given in Table 1 and in tables to follow.

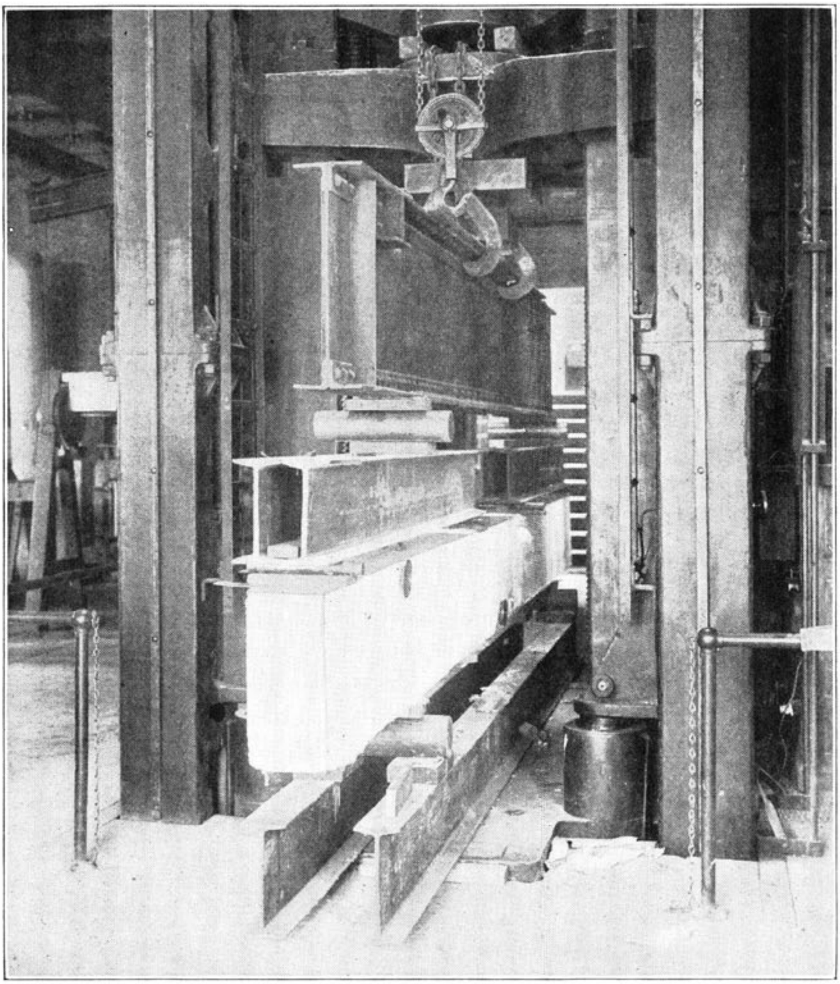


FIG. 3. VIEW OF BEAM IN TESTING MACHINE, SERIES OF 1911

7. *Reduction of Data.*—The strain gage readings were all reduced to unit deformations and then to unit stress, assuming the modulus of elasticity for the steel to be 30 000 000 lb. per sq. in. It is obvious that when the proportional limit was exceeded the stresses calculated from strain measurements are in error; this must be remembered when use is made of load-stress relations for members that have been stressed to or beyond the yield point.

It should also be noted that the strains measured are the average strains over a 4-in. or 6-in. gage line. In a bar crossed by a tension

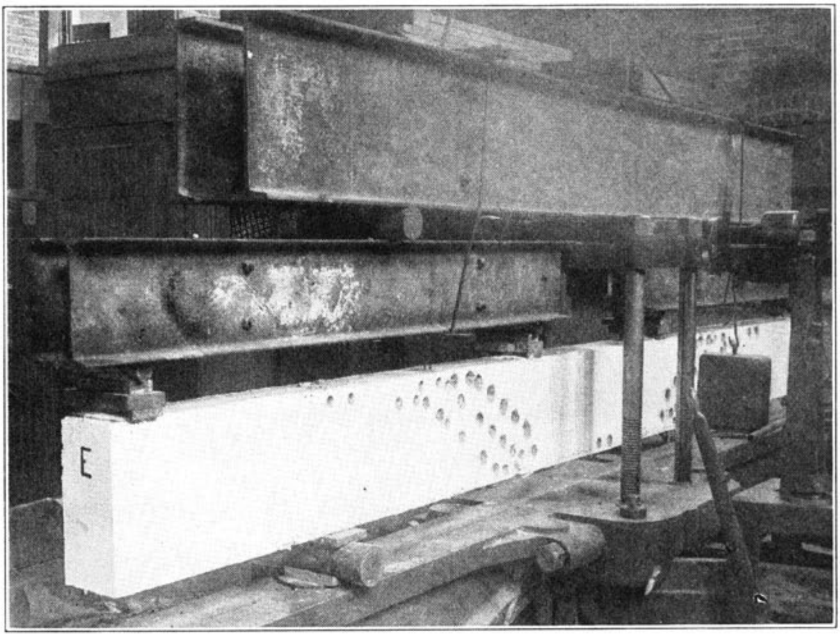


FIG. 4. VIEW OF BEAM IN TESTING MACHINE, SERIES OF 1917

crack the maximum stress may be considerably greater than the average stress over the gage line, due to the tension in the unbroken concrete adjacent to the crack. The variation in stress in the steel must be developed by bond and thus is limited by the bond resistance available. It appears that at an average stress of 20 000 lb. per sq. in. on a 4-in. gage length of a $\frac{5}{8}$-in. round bar used for web reinforcement the maximum stress might be 10 per cent greater than the average; for lower stresses, smaller bars, and longer gage lengths, the difference may be considerably larger. When stresses approaching the yield point are reached, however, cracking of the concrete is usually so general that the variation of stress within a gage line becomes relatively unimportant.

Methods of calculating other data of the tests should be noted. The percentage of longitudinal reinforcement was calculated from the cross-section of the beam and of the longitudinal steel at both support and mid-span. Since the moment at the support was twice as great as that at mid-span, the maximum longitudinal stress was generally found at the support. The calculated stress in the longitudinal steel was computed from the well-known equation $f_s = \frac{M}{Ajd}$, wherein M is the bending moment at the section considered, A is the cross-sectional area of the longitudinal steel, jd is the lever arm of the resisting couple, and d is the effective depth of the beam. Values of j were found as described in Section 11, Bulletin 166.

The nominal shearing unit stress and the nominal bond stress were calculated by means of equations (1) and (2), using the values of j found as noted above. The ratio of web reinforcement, r, has been calculated as the ratio of volume of web reinforcement to volume of concrete web reinforced. Due to the difficulty of interpreting the extent of their effectiveness, the inclined bars have been omitted in calculating the web reinforcement of the 1911 series.

In all cases the values of the maximum externally applied load have been given in the tables; however, in calculating tensile, shearing, and bond stresses, the weight of the beam has been included in the total load.

III. Tests of Series of 1911

8. *Description of Tests.*—As noted previously, the investigation of restrained beams was begun in 1911. Seventeen test beams were made, of which eleven were tested when about sixty days old and the remaining six in 1913 when they were about two years old.

The test beams had the general proportions, and the amounts, kinds, and arrangements of reinforcement that are found in beams in practice. The ratio of longitudinal reinforcement varied from 0.007 to 0.014, while the ratio of web reinforcement varied from 0.003 to 0.008. The arrangement of the reinforcement in the beams and the principal data of the tests are given in Table 4. The table shows that ten of the beams had loose vertical stirrups and longitudinal bars of mild steel, four had fabricated reinforcing units consisting of rigidly attached stirrups of mild steel and longitudinal bars of high carbon steel, and two had fabricated reinforcing units in which both the rigidly attached stirrups and the longitudinal bars were of high carbon steel. In all beams the bent-up portion of the longitudinal bars passed through the section of contraflexure.

The beams were loaded in testing as indicated in Fig. 2. Strain measurements were taken at each increment of load on a large number of 4-in. and 6-in. gage lines on longitudinal and web reinforcement. In these tests the conditions of loading were favorable to the development of high diagonal tension stresses in the vicinity of the supports. The bending moment was greatest at the support and the vertical shear was constant between the outer and inner load points. As might be expected, high diagonal stresses were found near the supports and

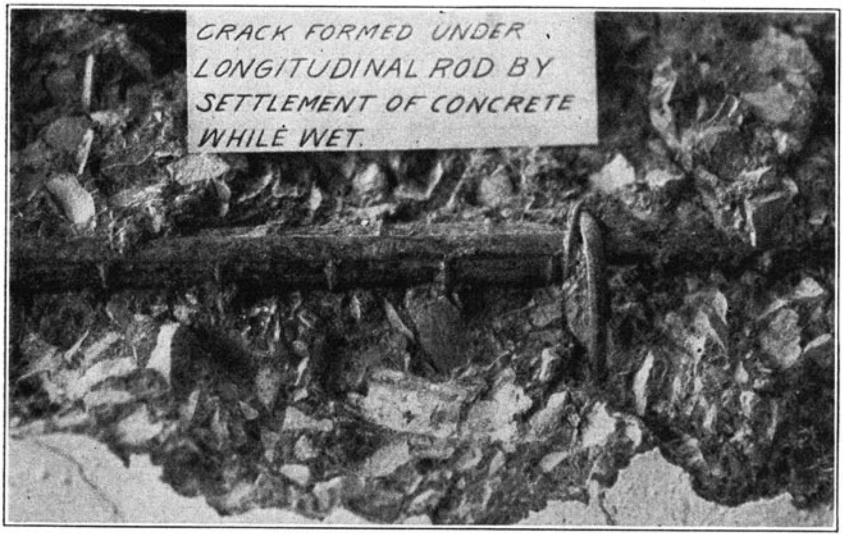
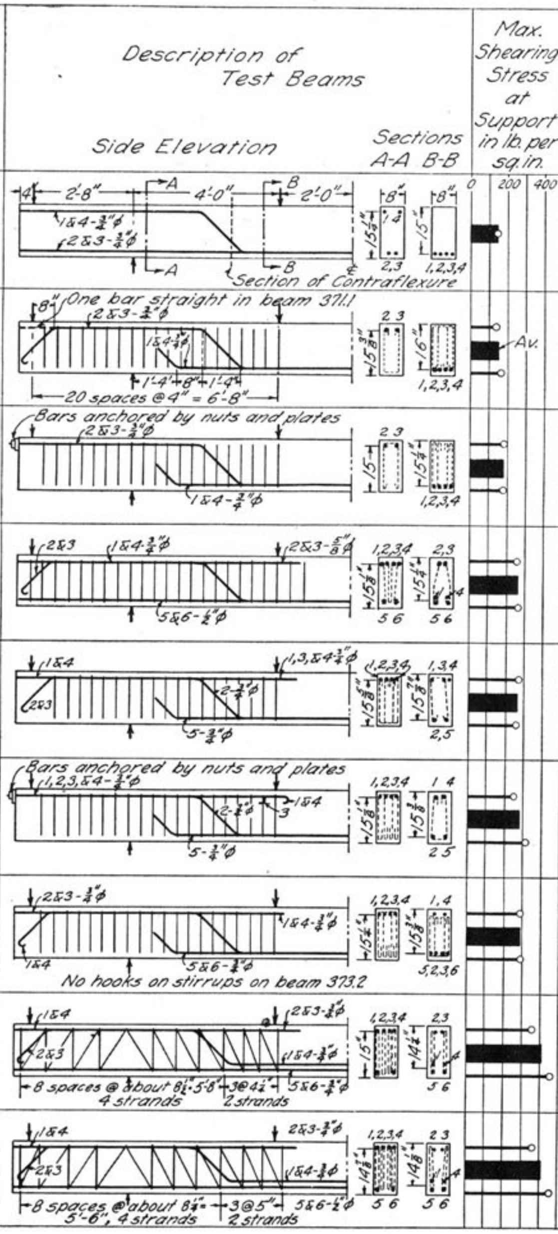


FIG. 5. VIEW SHOWING CRACK UNDER LONGITUDINAL BAR OF BEAM 376.2, DUE TO SETTLEMENT AND SHRINKAGE OF CONCRETE IN SETTING

numerous diagonal cracks formed in this region. Failure generally occurred by tension and bond at the support or just outside the support in the overhanging portion of the beam; evidence of diagonal tension failure was found in only four beams and even in these beams slipping of longitudinal bars was apparently the primary cause of failure. Measurements were taken of the slip of bars throughout the tests except on beams 371.1, 371.2, 375.1, and 378.1. The measurements were taken either at the ends or over the supports of each beam. In a number of instances early slipping of the unanchored horizontal bars occurred and brought about high stresses in the anchored bars and in the web reinforcement and correspondingly lower stresses in the bars that slipped. Cracks or cavities caused by the settlement during setting of the freshly placed concrete from beneath the negative reinforcement were found in several cases; they obviously reduced the bond between the concrete and steel, and may have brought about the serious slipping observed at low loads. Figure 5 shows a view of a settlement crack found by breaking into a beam after test. In the beams having bars anchored by plates and nuts, local slipping was noted near the support but the anchorage prevented early failure in bond. In other beams crushing of the concrete at the bends and inside the hooks of longitudinal bars occurred at high loads. In some of the beams reinforced with deformed bars, crushing of the concrete was

TABLE 4

PRINCIPAL RESULTS OF TESTS OF RESTRAINED BEAMS, SERIES OF 1911



Beam No.	Manner of Failure
374.1	Bond & Tension
371.1	Bond & Tension
371.2	Tension
378.1	Bond & Tension
378.2	Bond & Tension
375.1	Bond & Diag. Tension
375.3	Bond & Diag. Tension
372.1	Tension
372.2	Bond & Tension
377.1	Bond & Tension
377.2	Bond & Tension
373.1	Tension
373.2	Tension
376.5	Bond & Tension
376.6	Tension
376.1	Bond & Diag. Tension
376.2	Tension, D.T., & Bond

4-1

TABLE 4 (Concluded)

PRINCIPAL RESULTS OF TESTS OF RESTRAINED BEAMS, SERIES OF 1911

Beam No.	Age at Test, Days	Reinforcement					Maximum Applied Load, lb	Computed Stresses lb per sq. in.						Cylinders Compress. Strength, lb. per sq. in.	
		Longitudinal			Web*			Longitudinal Steel		Shearing Stress		Bond		Stored:	
		Kind	Per Cent Support	Per Cent Center	Kind	%		Support	Center	Support	Center	Support	Center	In Sand	With Beam
374.1	61	Pl. ϕ m.s.	0.74	1.48	None	—	57100	39500	11400	142	146	242	124	254	1630
371.1	731	Pl. ϕ m.s	0.72	1.38	¼" pl. ϕ, U-stirrups, 4" Space.	0.31	57100	39300	10700	141	137	241	116	—	—
371.2	62		0.72	1.44		0.31	65400	44800	12100	161	156	272	132	303	2550
Av.										151	146	257	124		
378.1	717	Pl. ϕ m.s.	0.72	1.44	¼" pl. ϕ, U-stirrups, 4" Space	0.31	75000	51600	14600	184	190	313	161	—	—
378.2	735		0.73	1.44		0.31	74300	51100	14400	182	188	309	160	—	—
Av.										183	189	311	160		
375.1	73	Pl. ϕ h.c.s.	1.25	1.06	¼" pl. ϕ, Stirrups, Rig. att., 4" space.	0.31	102100	43700	26100	262	249	242	254	440	1716
375.3	790		1.25	1.03		0.31	103500	44300	26400	265	253	245	257	—	—
Av.										264	251	244	256		
372.1	68	Pl. ϕ m.s.	1.44	0.72	¼" pl. ϕ, U-stirrups, 4" Space.	0.31	110000	38600	37900	272	252	231	427	338	2030
372.2	64		1.44	0.72		0.31	103600	36400	35800	257	238	213	402	318	2230
Av.										265	245	225	414		
377.1	731	Pl ϕ m.s.	1.44	0.70	¼" pl. ϕ, U-stirrups, 4" space	0.31	96600	35000	34500	247	228	210	387	—	—
377.2	721		1.46	0.74		0.31	124500	45000	44000	316	293	268	498	—	—
Av.										282	261	239	443		
373.1	65	Pl. ϕ m.s.	1.44	1.44	¼" pl. ϕ, U-stirrups, 4" space	0.31	114000	41000	21200	289	281	245	239	244	2130
373.2	60		1.44	1.44		0.31	116000	41700	21500	294	286	250	243	310	2680
Av.										292	284	248	241		
376.5	61	Cor. ϕ h.c.s.	1.45	1.44	¼" cor. ϕ, Rigidly att'ch'd See elevation.	0.80	139800	51000	27800	358	371	304	316	—	1600
376.6	64		1.45	1.44		0.80	180000	65600	35400	458	477	390	406	436	2510
Av.										408	424	347	361		
376.1	67	Cor. ϕ h.c.s.	1.47	1.13	0.21" cor. ϕ, Rigidly att'ch'd. See elevation.	0.46	141100	52000	38600	366	373	310	379	294	1520
376.2	67		1.47	1.13		0.46	178000	65600	48300	459	470	390	478	265	2020
Av.										412	422	350	428		

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*Inclined bars not included as web reinforcement.

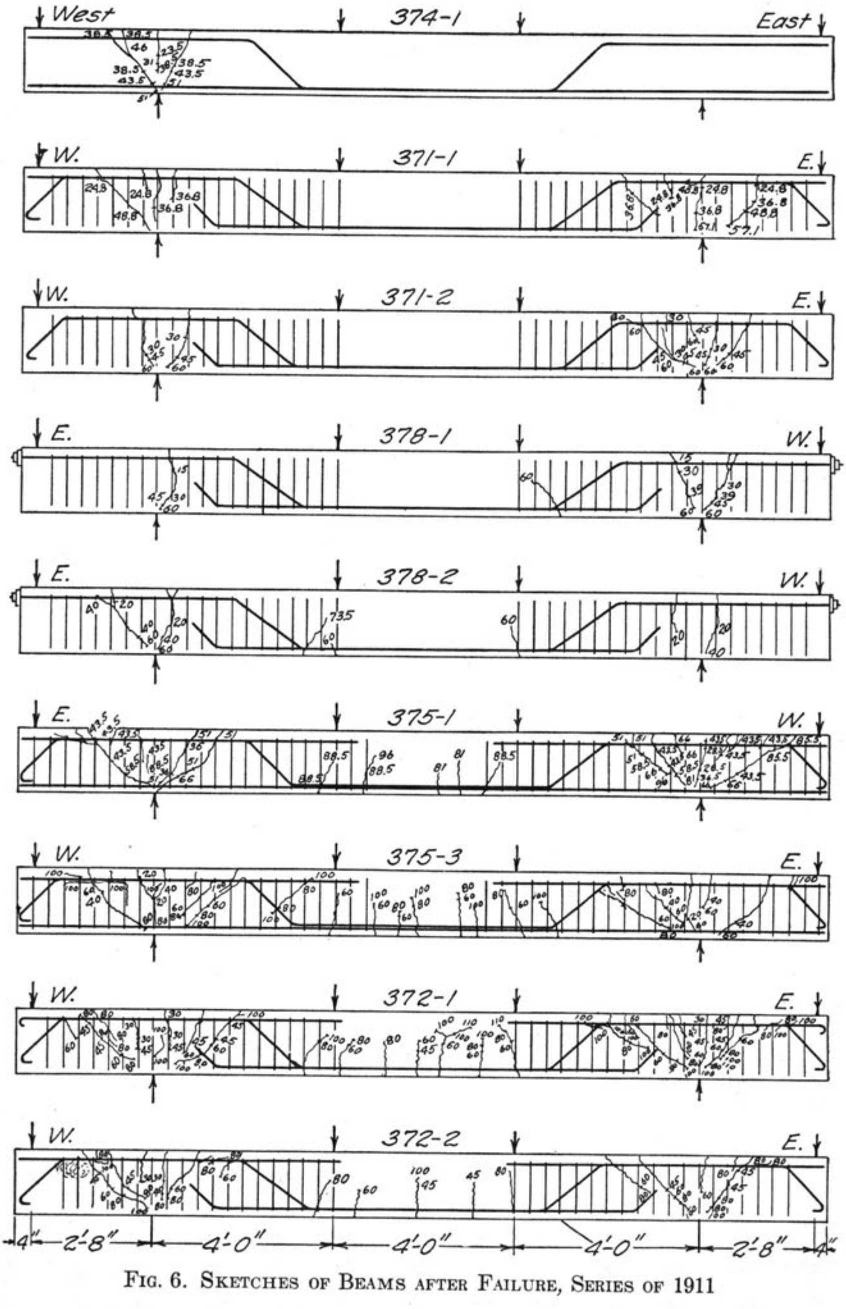


FIG. 6. SKETCHES OF BEAMS AFTER FAILURE, SERIES OF 1911

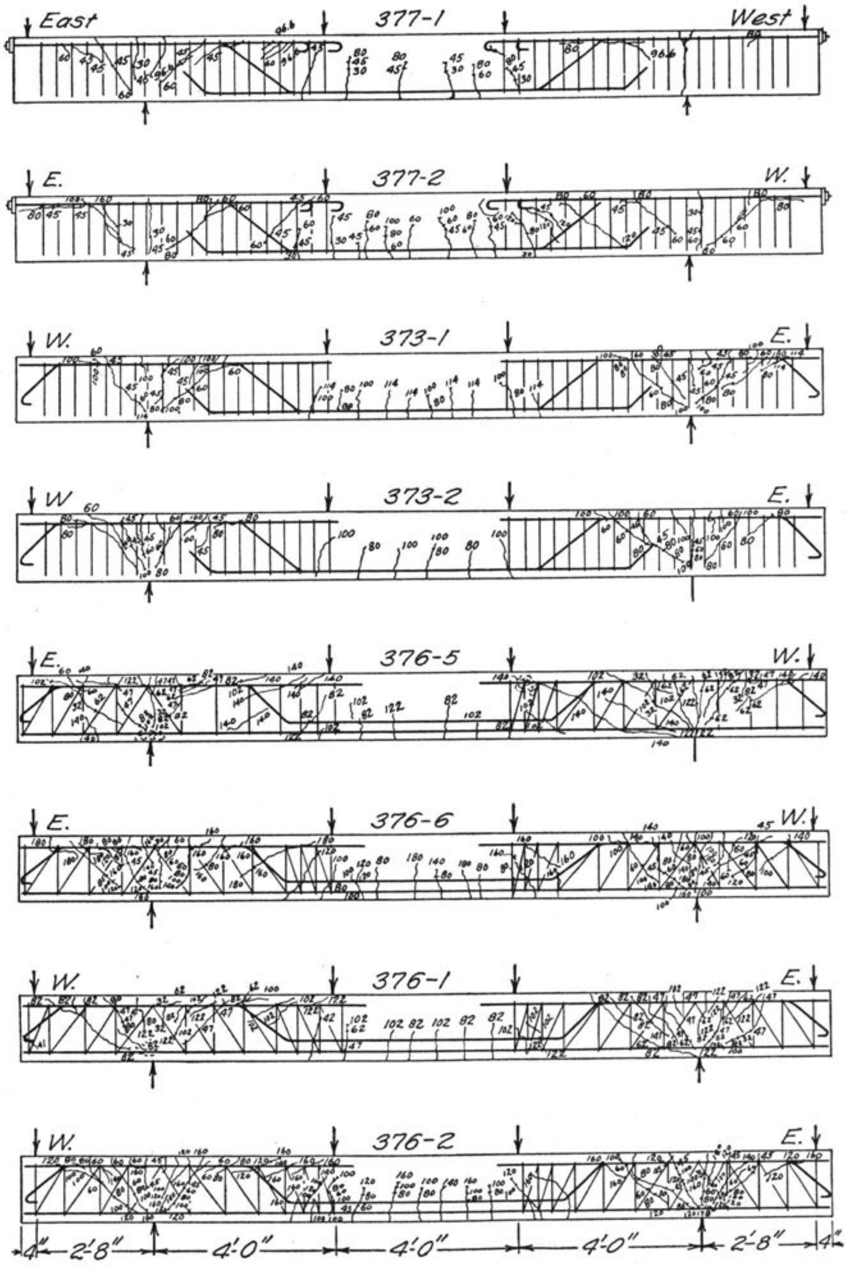


FIG. 7. SKETCHES OF BEAMS AFTER FAILURE, SERIES OF 1911

noted in front of the projections on the bars. As had been found in previous tests, little stress was produced in stirrups until they were crossed by diagonal cracks. After the formation of cracks the stress in stirrups increased rapidly and in a number of cases it exceeded the yield point strength of the stirrups.

9. *Phenomena of Tests.*—Detailed notes of the tests may be found useful in interpreting the data given in Table 4. Figures 6 and 7 will also give an idea of the manner of failure, and the location of cracks with respect to the position of the reinforcement and of the loads and reactions. The extent of a crack at various loads is indicated by a cross mark opposite the numeral representing the total load in thousands of pounds.

Beam 371.1. In this beam one of the two ¾-in. bars over the supports was bent down at an angle of about 45 deg., 12 in. from the end of the beam and was hooked at the end. The other longitudinal bar was continued straight to the end of the beam. At a load of 36 800 lb. these bars showed equal strains on corresponding gage lines, but at the load of 48 000 lb. the strain readings showed that the anchored bar at the west end was more highly stressed than the unanchored bar, indicating that the latter was slipping. Failure occurred at the load of 57 100 lb. by tension in the anchored bar at the west end of the beam due to slipping of the unanchored bar. After the test it was found that the unanchored bar had slipped about $3/16$ in. No slipping of the anchored bar or of the stirrups was noted. Settlement cracks were observed under the longitudinal bars at various points. Some of the stirrups at the west end of the beam were stressed to the yield point at the maximum load.

Beam 371.2. This beam was similar to beam 371.1 except that both longitudinal bars in the overhanging ends were bent down at an angle of 45 deg., 12 in. from the end of the beam and anchored by hooks. The beam failed by tension in the horizontal steel over the supports at a load of 65 400 lb. No movement of the anchored ends of the longitudinal rods was visible after the test but several settlement cracks were found under the longitudinal bars. The stresses observed in the stirrups were low.

The middle part of the beam was retested as a simple beam on a 9-ft. span with loads applied at the one-third points. It failed by tension in the longitudinal steel at a load of 49 000 lb., corresponding to a shearing unit stress of 234 lb. per sq. in.

Beam 372.1. This beam failed by tension in the longitudinal steel near the yield point at the maximum load of 110 000 lb. Slip of the ends of the bars at the west end of the beam was first observed at a load of 80 000 lb., and had increased to over 0.1 inch at a 100 000-lb. load. There were indications that the corresponding bars at the east end had slipped a corresponding amount. A number of settlement cracks under the horizontal bars over the supports were noted. High stresses were observed in a few of the stirrups which were crossed by diagonal cracks.

Beam 372.2. At the maximum load of 103 600 lb. the steel at the middle of the beam had passed the yield point and the steel over the supports was also highly stressed. Measurements on the anchored bars indicated that they were stressed to the yield point. The first observed slip of 0.0018 in. of the unanchored bars at the end of the beam was noted at a load of 80 000 lb. and thereafter the beam took load slowly. Crushing of the concrete occurred on each side of the beam under the bends in the longitudinal bars and this together with the slipping of the unanchored bars caused the diagonal cracks at the west end of the beam to open about ⅛ in. at the final stage of the loading. The concrete immediately under the bends was found to be crushed to a powder. The slipping of unanchored bars at the end of the beam caused high stresses to be developed in the anchored bars. In some of the stirrups outside the west support the yield point of the steel was passed.

Beam 373.1. Failure occurred at a load of 114 000 lb. by tension in the steel over the supports. Slipping of the unanchored bars at the end of the beam began at loads of 80 000 to 90 000 lb., producing lower tensile stresses in these bars than in the anchored ones. Stresses ranging from 20 000 to 28 000 lb. per sq. in. were observed in stirrups near the section of contraflexure.

Beam 373.2. At a load of 116 000 lb. failure occurred by tension in the steel over the supports. Slipping of the horizontal bars was not so serious in this beam as in beam 373.1, the maximum slip observed being 0.003 in. at a load of 100 000 lb. High stresses were observed in the stirrups crossed by cracks.

Beam 374.1. There were no stirrups in this beam. The percentage of longitudinal reinforcement was twice as great at mid-span as at the supports. At a load of 38 500 lb. three cracks had formed, a vertical one directly over the west support and a diagonal one on either side extending toward the support. Failure occurred at a load of 57 000 lb. due to tension in the steel over the support; slips of 0.0004 to 0.0010 in. were also measured on the steel at the ends of the beam. Although the diagonal crack in the overhanging end was large at failure it closed up upon release of the load.

The middle part of the beam was retested as a simple beam, with 6-ft. span and one-third point loading; it developed a shearing unit stress of 293 lb. per sq. in. (maximum load 61 000 lb.) when failure occurred by diagonal tension.

Beam 375.1. This beam, which contained a fabricated reinforcing unit, failed by bond and diagonal tension. At the maximum load of 102 100 lb. large diagonal cracks had opened outside the supports. No measurements of slip of bars were taken during the test, but by cutting away the concrete to expose the unanchored bars over the support after the test it was found that they had slipped as much as 0.15 in. This slipping transferred high stresses into the anchored bent-down bars outside the support, probably reaching the yield point at the maximum load.

Beam 375.3. At a load of 60 000 lb. a slip of unanchored bars of 0.001 in. was noted, and as the load increased to 100 000 lb., the slip increased to 0.143 in. Failure occurred primarily by bond at a load of 103 500 lb. The unanchored bars were relieved of stress when slipping occurred and high stresses were induced in the anchored bars. Strain readings showed that one stirrup outside the west support was stressed to the yield point.

Beam 376.1. This beam was reinforced with a fabricated unit having high carbon longitudinal and mild steel web members. Slipping of unanchored straight bars was first noted over both supports at a load of 47 300 lb., and at a load of 60 000 lb. had increased to 0.001 in. At a load of 120 000 lb. this slip had increased to 0.045 in. At the maximum load of 141 100 lb., a large diagonal crack opened outside the west support and crushing of the concrete near the support followed. The stresses in the longitudinal steel did not reach the yield point. High stresses were found in both vertical and inclined stirrups near the support.

Beam 376.2. Failure occurred at a load of 178 000 lb. The two anchored bars over the west support were stressed to the yield point, but due to slipping the stress in the unanchored bars did not reach the yield point. Several stirrups were stressed to the yield point at the maximum load. Failure occurred by tension, diagonal tension, and bond, the measured slips being similar in amount to those observed in beam 376.1. Numerous settlement cracks which were found under the horizontal bars over the supports greatly reduced the area available for bond.

Beam 376.5. This beam was reinforced with a fabricated unit in which the inclined and vertical stirrups consisted of ¼-in. square corrugated bars of high carbon steel, giving a ratio of web reinforcement of 0.008. Failure occurred at a load of 139 800 lb. Slipping of the unanchored straight bars at the west end of the beam apparently was the primary cause of failure. The first measured slip of 0.0015 in. was noted at a load of 80 000 lb. At the maximum load these bars were found to have slipped from 0.21 to 0.52 in. The slipping of these bars induced high stresses in the anchored bars and they were stressed to the yield point. At the maximum load the concrete crushed near the inner load point at the west end of the beam and near both supports. A maximum stress of 25 600 lb. per sq. in. was observed in the stirrups.

Beam 376.6. The reinforcing unit was similar to that in beam 376.5. Failure occurred by tension in the steel over the supports at a load of 180 000 lb. Some slipping of the unanchored straight bars over the supports had occurred at failure. An examination after the test showed that crushing of the concrete had occurred at the corrugations on the reinforcing bars. Inclined stirrups at the west end of the beam were stressed nearly to the yield point at the maximum load.

Beam 377.1. The longitudinal bars were anchored at the ends of the beam by means of nuts and washers bearing against steel plates. The nuts were tightened just before load was applied to the beam. At a load of 30 000 lb. a tension crack formed over the west support extending downward past mid-height of the beam; at failure this crack opened very wide. The first slip of the bars over the supports was measured at this load and was equal to 0.0038 in. In the vicinity of the support uniform longitudinal stresses were noted over a considerable length of bar. At maximum load there was little bond between these bars and the surrounding concrete, anchorage being afforded by the hooks at the inner ends of the bars and the nuts and washers at the outer ends. Failure occurred at the load of 96 600 lb. due to bond and tension. A maximum stress of 24 000 lb. per sq. in. was observed in the stirrups.

Beam 377.2. Slipping of the two outer longitudinal bars over the support began at a load of 30 000 lb., the amount of the slip being 0.003 in. At a load of 45 000 lb., the measured slip of bars had reached 0.001 in. The stresses in the stirrups of this beam, at the maximum load of 124 500 lb., were high, ranging from 12 000 lb. per sq. in. to the yield point of the steel. Stirrups near the section of contraflexure developed stresses as great as 22 000 lb. per sq. in. Failure was due to bond, followed by tension in the steel.

Beam 378.1. The two bars over the supports in this beam were anchored at the ends of the beam by means of nuts and washers bearing against steel plates. Slip measurements were not taken on this beam. Failure occurred at a load of 75 000 lb. due to bond followed by tension in the steel over the support. The greatest stress observed in the stirrups was 34 000 lb. per sq. in. The strain readings indicated that beyond a load of 35 000 lb. there was lack of bond between the longitudinal rods and the concrete outside the west support.

Beam 378.2. The first slip of the bars over the support was noted at a load of 20 000 lb. With increasing loads the nuts and washers became effective and prevented early failure by bond. Only where cracks formed did the stirrups receive high stresses. In some instances the stirrups were stressed to the yield point at the maximum load of 74 300 lb. Failure was due to bond, followed by tension in the steel over the support. The slip of bars over the support at failure amounted to 0.067 in.

10. *Web Resistance of Restrained Beams.*—The calculated values of the shearing unit stress for the beams of this series of tests are given in Table 4. The values range from 142 lb. per sq. in. for beam 374.1, which had no web reinforcement, to 459 lb. per sq. in. for beam 376.2, which was reinforced with a fabricated unit. The values of shearing stress which will be considered are those calculated for a section near the support where failure generally occurred. Although in some cases the shearing stresses at other sections were slightly higher due to variations in effective depth of beam, the measured stirrup stresses at these points were generally low, and hence these were not critical sections.

Beams 376.1, 376.2, 376.5, and 376.6, reinforced with fabricated units, developed shearing stresses considerably higher than the others of the series. In these beams diagonal cracks in the region about the supports and between the supports and inner load points were numerous, and high stresses were developed in the stirrups at the maximum load, as well as in the concrete and the longitudinal steel; it was sometimes difficult to determine the primary cause of failure. The longitudinal reinforcement in these beams consisted of deformed bars of high carbon steel and in the two last mentioned the web reinforcement was also of high carbon steel. This group of beams had a larger percentage of web reinforcement than any of the other beams of the series.

11. *Stresses in Web Reinforcement.*—The measured stresses in typical stirrups at various increments of load for the different beams of the series have been plotted in Fig. 8. Each curve represents the stress on a certain gage line, generally just inside or just outside the

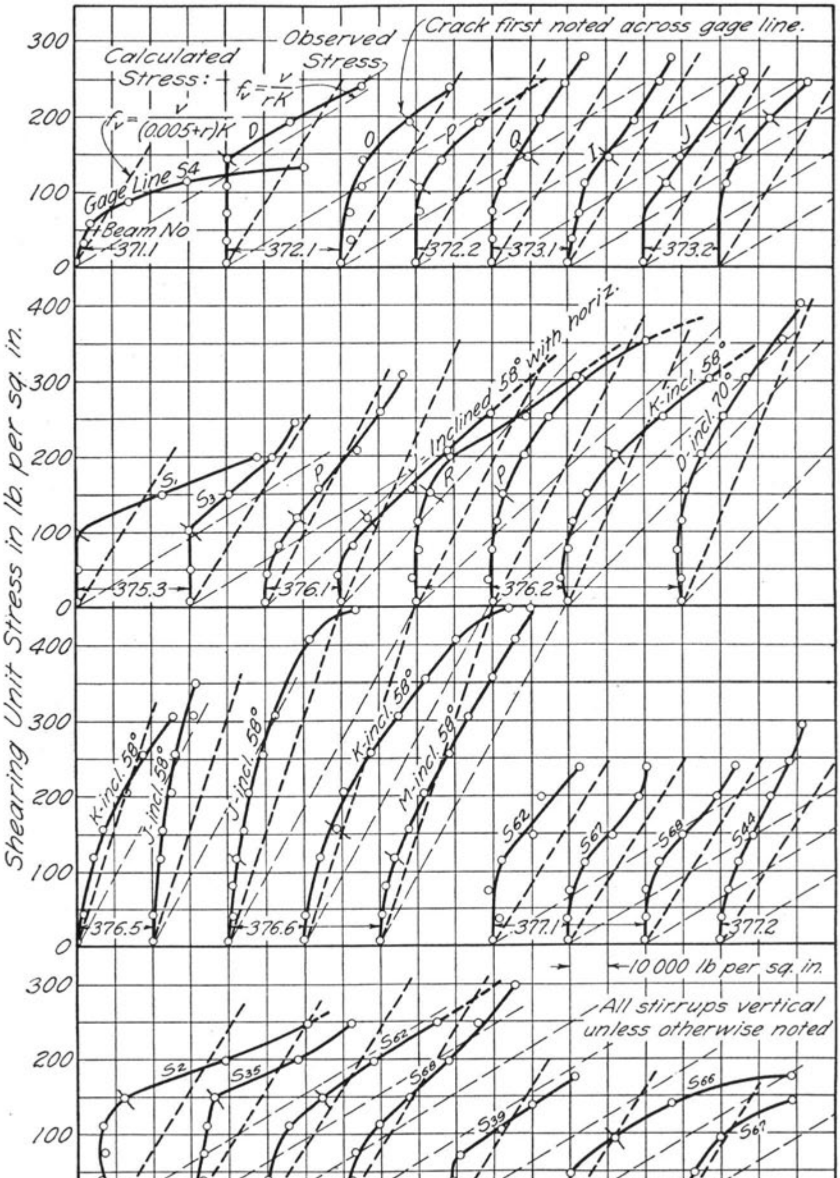


Fig. 8. Load-stress Curves for Web Reinforcement, Series of 1911

support, in the region where the greatest stirrup stresses were observed and where diagonal cracks were most numerous.

The curves of Fig. 8 are of the usual type for web members, showing little stress until after the formation of a crack across the stirrup, then a rapid increase in stress as the load is increased. The stress apparently depends greatly on the way in which a crack chances to intersect the stirrup. While a marked similarity of action was noted on the two sides of a beam, as in the two prongs of a stirrup, such similarity was not generally found at corresponding sections at the two ends of the beam. Cracks formed in different positions at the two ends and the variations in stress distribution in longitudinal bars near the supports due to slipping produced irregular conditions in the stirrups, especially in the overhanging ends of the beam.

The load at which the first crack intersected the stirrup is indicated in Fig. 8. A study of similar load-stress curves for all of the beams leads to the conclusion that the load at which cracks opened was a function of the percentage of longitudinal steel at the support. The load at which the stirrups were first stressed appreciably was generally greatest in the beams having 1.45 per cent of reinforcement over the support and least for those having 0.72 per cent. The increase in percentage of steel apparently inhibited the formation of cracks and hence increased the load at which the stirrups came into action. It may be noted also that the maximum loads carried were largely a function of the percentage of longitudinal reinforcement at the support, which is consistent with the way in which the failures occurred.

To compare with the measured stresses in stirrups shown in Fig. 8, calculated stresses based on the use of the full shearing stress according to equation (5) are also shown in the figure by light broken lines. It should be remembered in considering the measured stresses that for the high carbon steel stirrups of beams 376.5 and 376.6 the yield point strength was about 60 000 lb. per sq. in.; for the others it did not exceed 40 000 to 45 000 lb. per sq. in.

A comparison shows the measured stresses to be considerably less than the calculated stresses in all cases, although the agreement is much better at high loads than at low ones. For a number of the vertical stirrups, after the formation of cracks the curves of measured stress seem to become nearly parallel to those representing calculated stresses; others show less agreement. The inclined stirrups used in beams 376.1, 376.2, 376.5, and 376.6, in combination with vertical stirrups, seem to show a fair agreement between measured and calculated stresses at all loads; they apparently took some stress at low loads. In stirrup K, beam 376.6, the observed stress equalled the calculated stress at the maximum load.

A second comparison may be made between the observed stress shown in Fig. 8 and the stresses calculated by use of equation (9), which are represented by heavy, dotted lines. A divergence between measured stresses and stresses calculated by means of equation (5) has generally been observed in previous tests, and equation (9) represents an attempt to express the variation as a function of the percentage of web reinforcement. For stirrups inclined 45 or 90 deg. to the horizontal, the ratio between the stress calculated by use of equation (9) and the stress calculated by assuming the entire shear to be carried by the web reinforcement may be expressed by the quantity $\left(\frac{r}{0.005 + r}\right)$, and for other angles of inclination it appears that the same ratio should obtain (see equation (11), Section 18). For ratios of web reinforcement of 0.0031, 0.0046, and 0.0080, the corresponding values of the stress ratio become 0.38, 0.48, and 0.62, respectively.

It appears that the curves of Fig. 8 based on equation (9) agree much better with the curves for measured stresses than do the curves based on equation (5). Equation (9) is intended to apply only at loads near the maximum; for such loads agreement would be shown by an intersection of the upper ends of the curves for measured and calculated stresses. In general, the values of the maximum measured stress average slightly greater than those calculated from equation (9), but since these stresses are among the highest observed in the several beams it may be said that the equation gives fairly accurately the stresses in stirrups at loads approaching the maximum. For low loads it gives values that are too high.

12. *Variations in Stirrup Stress Along Beams.*—The results of the strain measurements taken at close intervals along the length of several beams show that the stirrups most highly stressed were located a distance on either side of the support approximately equal to the depth of the beam. Higher stresses were found in the stirrups outside of the support than in those in the region between the support and the inner load point. As was previously stated, these regions of high stirrup stress were coincident with the regions in which diagonal cracks were large and numerous.

To show the variation in stirrup stresses along the length of a beam, Fig. 9 has been plotted. It shows values of the stresses measured on two gage lines on each of a large number of stirrups in four beams. The readings were taken at one end of the beam, gage lines on alternate stirrups being on one side of the beam and the remainder on the opposite side; this may account for some variations in the stirrup stresses from point to point. The stirrup stresses shown are for different loads on the four beams, each being at or near the maximum load carried by the beam. It is seen that high stresses were developed on only a few gage lines; on the majority of the gage lines there was little stress in the stirrups. The stresses were quite low near the load points, a slight compression being produced in some stirrups at the load points by the high local bearing stresses developed.

The stresses observed in stirrups near the section of contraflexure or between this section and the inner load point were generally quite low. Exceptions to this rule were found in beam 376.2 in which a stress of 31 000 lb. per sq. in. was measured in an inclined stirrup near the inner load point at a shearing unit stress of 400 lb. per sq. in., and in beam 377.2, Fig. 9, in which stirrup stresses as high as 19 000 lb. per sq. in. were observed near the section of contraflexure at a shearing unit stress of 293 lb. per sq. in. In the other beams tested the stirrup stresses at similar sections were negligible.

Strain measurements taken on the inclined portion of bent-up bars at the section of contraflexure showed that considerable stress was developed in these bars, which were evidently effective in resisting diagonal tensile stresses in this section of the beam. Stresses measured near the maximum load of several beams range from 15 000 lb. per sq. in. in beam 371.1 to 27 000 lb. per sq. in. in beam 376.1 and 33 000 lb. per sq. in. in beam 378.1. The tests do not indicate, however, just how great a region may be reinforced against diagonal tension by bars bent up in a single plane.

13. *Effect of Slipping of Longitudinal Bars.*—Frequent mention has been made of the slipping of longitudinal reinforcement at the supports and in the cantilever ends of the beams. This slipping was measured at the ends of the straight bars that extended to the ends of the beam; in some of the beams slipping of the bars at sections between the support and the load points was also measured. This slipping was an important factor in the failure of many of the beams. In Table 4, several beams are listed as having failed by bond or by a combination of bond and tension or diagonal tension. The slipping of bars was evidently due to a progressive bond failure starting in the region of high tensile stress near the support and extending toward the ends of the reinforcing bars. It is quite evident from the variation in measured stress that the intensity of bond stress varied considerably and was frequently much greater than the nominal bond stress

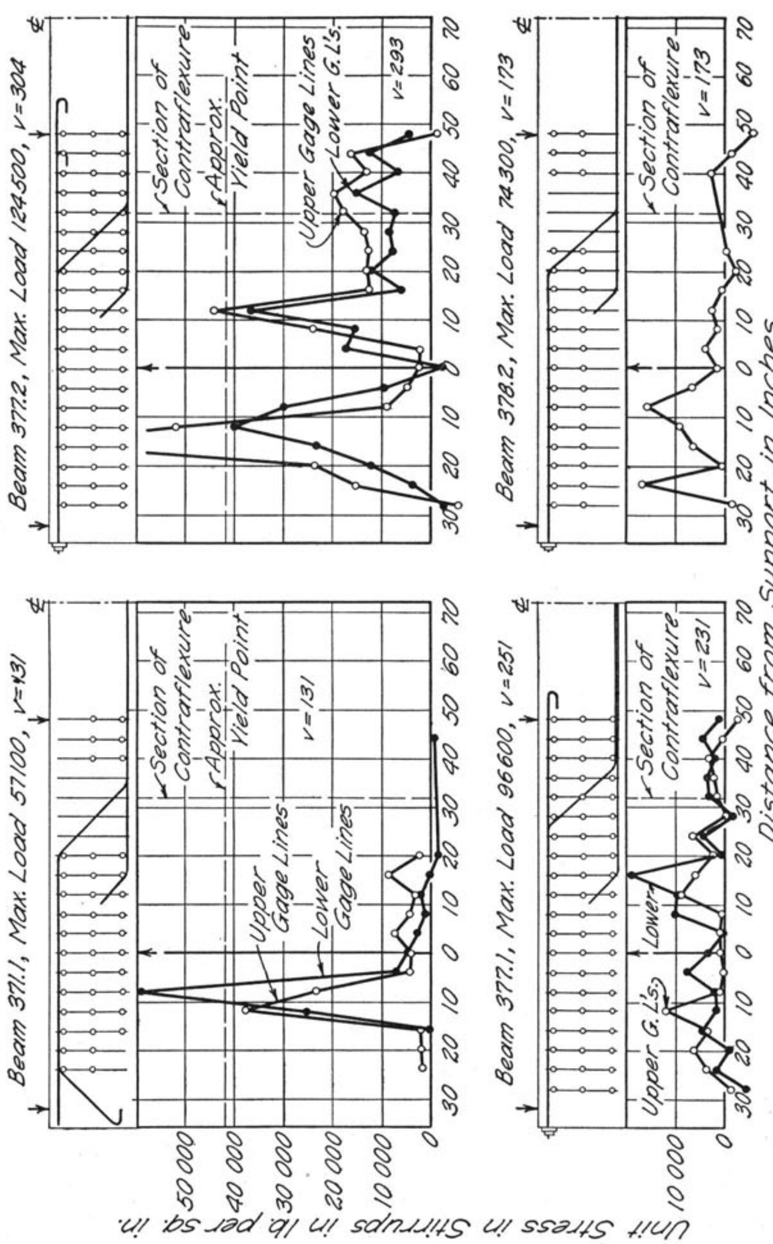


FIG. 9. MEASURED STRESSES IN STIRRUPS IN TYPICAL BEAMS, SERIES OF 1911

given by equation (2). Generally in these beams two bars were run straight over the support to the end of the beam and two were bent down and anchored by hooks near the end of the beam. Slipping of the unanchored bars occurred at relatively low loads; in consequence, an undue proportion of the total stress was shifted to the anchored bars, thus producing high deformations and numerous cracks. The result was to cause larger deformations in the concrete web and to produce larger stresses in stirrups and inclined bars than if no slipping had occurred. A close relation was observed in the tests between the beginning of slip, the formation of diagonal cracks, and the development of stirrup stresses. The observations made during these tests indicate that in various published records of tests bond may have played a considerable part in failures that are classed as due to diagonal tension.

Measurements of slip indicated that initial slipping of plain round bars and of deformed bars occurred at about the same load. The rate of subsequent slipping was generally greater for the plain bars than for the deformed ones. The results are masked, however, by the presence of settlement cracks beneath the bars, as shown in Fig. 5, which undoubtedly reduced the effective bond area of the bars considerably. In some cases it appears that as much as one-third of the bond area of the bar may have been lost. In any case it is reasonable that the deformed bars should give somewhat higher resistance to large amounts of slip than the plain bars. Crushing of the concrete in front of the corrugations or projections of the deformed bars was noted after the tests in all of the beams having corrugated bar reinforcement.

In contrast to the slipping of longitudinal bars just described, no slipping of stirrups was observed in any of the beams tested. This is evidently due to the small diameter (¼ in. or less) of the stirrups and the adequate anchorage and embedment provided at the ends of the stirrup prongs. Even in beam **373.2**, in which the ends of stirrups were not hooked, sufficient embedment was available to prevent slipping. In some of the beams a space was purposely left between stirrup loop and longitudinal bar to see if the intervening concrete would be crushed when the stirrup was highly stressed. No crushing of this sort was noted, the inference being that very high bearing pressures may be sustained on a small area of concrete that is well restrained by the surrounding mass of the concrete web.

14. *Anchorage of Longitudinal Bars.*—Anchorage of longitudinal bars was secured in several ways: (*a*) by providing a hook, as seen on the upper reinforcing bars of beams **377.1** and **377.2**, near the inner

load points; (*b*) by bending the bar down into the compression side of the beam and providing a hook at the end; and (*c*) by threading the end of the bar, which was allowed to project beyond the end of the beam, and anchoring it by means of nuts and plates. The first method was not very satisfactory since in the tests of beams 377.1 and 377.2 the bars slipped considerably, and at the maximum load the concrete crushed inside the hooks, although this crushing and splitting out of the concrete was partly due to the fact that the bars were located too near the lateral concrete surface and were bent to a very small radius. The second method, that of bending a bar down and hooking it in the compression side of the beam, gave very satisfactory anchorage. Some crushing was noted under bends and within hooks, which indicates that such details should be carefully designed to avoid high bearing stresses in the concrete. Furthermore, hooks and bends should be kept well away from the surface of the concrete in order to secure the desired lateral restraint or support around the area of high local bearing stress. The third method of anchorage, employing bearing plates and nuts which were tightened before the test, gave satisfactory results and in some cases prevented premature bond failure. The method is, of course, not suited to practical use in construction.

IV. Tests of Series of 1917

15. *Outline of Series.*—A study of the tests of restrained beams made in 1911 showed that there were many phases of web resistance in beams of this type on which information was needed. The 1917 series was planned with a view to securing information on a number of topics, including the feasibility of using bent-up bars alone as web reinforcement, the effect of the angle of inclination and the spacing of the inclined bars, the effect of the distance from the support to the first bent bar or stirrup, the distribution of web stresses along the portion of the beam subject to shear, and various allied questions.

The test beams were of the same overall dimensions as those of the 1911 series, being 18 ft. long, 8 in. wide, and 15 in. in effective depth. They were of 1:2:4 concrete and were tested at the age of 60 days. The reinforcement was of mild steel. Physical properties of the materials used are given in Tables 1, 2, and 3.

In an attempt to prevent failure by tension in the longitudinal steel or by slipping of bars due to insufficient anchorage, a larger amount of longitudinal reinforcement, and more carefully designed anchorage of the ends of bars, were used than in the 1911 tests. The longitudinal reinforcement consisted of eight ⅝-in. round bars (about

2.0 per cent) at the support and from four to eight ⅝-in. round bars (1.0 to 2.0 per cent) at mid-span. The bars were generally placed in two layers at sections of both positive and negative moment.

In designing the web reinforcement, care was taken to prevent web failure in the overhanging portions of the beam, since the investigation was concerned mainly with the effect of various arrangements of bent-up bars on the web resistance of the portions of the beam between the supports and the inner load points. The overhanging portions were reinforced by bending down the longitudinal bars and anchoring them by means of hooks; a few vertical stirrups were also used near the support. No failures occurred in these overhanging ends of the beams.

The principal web reinforcement for the region between support and inner load point was provided by bending down longitudinal bars at points where they were not required to provide resisting moment. Various arrangements of these bent-down bars were devised. In one group the distance from the support to the first bend was varied, in another group the angle of bend was the variable, and in another group different horizontal spacings between bends were used. In a few beams vertical stirrups were used in addition to the bent bars. In beams of three types the bent bars were placed in one layer, but in most of the beams the bars were bent down in two, three, or four layers or flights. One type of beam was made in which no web reinforcement was used in order to secure information on the web strength of the concrete. An outline of the series is given in Table 5, wherein the beams are placed in five principal groups, according to the different variables to be studied. Some of the beams are included in two different groups. There were 21 types of beams, each made and tested in duplicate.

Details of the reinforcement of all of the beams and the principal results of the tests are given in Table 6.

16. *Phenomena of Tests.*—The tests were featured by a large number of strain gage readings, taken on 4-in. gage lines on the longitudinal and web reinforcement (see Figs. 12 to 19). Loads were applied to the beams so as to produce increments in shearing stress of 50 lb. per sq. in. or multiples thereof. Generally four to nine sets of strain measurements were made, the last set being taken when the steel had reached its yield point at some gage line or when failure seemed imminent. Slip of certain of the reinforcing bars was also measured in some of the first beams tested; these slips were so consistently small that such readings were omitted in the later tests.

TABLE 5

OUTLINE OF TESTS OF RESTRAINED BEAMS, SERIES OF 1917

Group	Beam No.	Distance from Support to First Bend, in.	Angle of Bend, deg.	Horizontal Spacing between Bent Bars, in.	Distance from Support to First Stirrup, in.	Total Number of Bent Bars		Remarks
						Bars	Layers	
	Effect of Distance from Support to First Bend.							
I.	386	8	32½	8	....	6	3	No Stirrups.
	391	12	32½	8	....	6	3	" "
	392	16	32½	8	....	6	3	" "
	393	8	45	8	....	6	3	" "
	394	12	45	8	....	6	3	" "
	395	16	45	8	....	6	3	" "
	Effect of Angle of Bend (see Group I, also).							
II.	382	8	22	12	....	5	2	No Stirrups.
	383	8	32½	12	....	6	3	" "
	384	8	45	12	....	6	3	" "
	385	8	32½	12	....	5	2	" "
	Effect of Spacing of Bars.							
III.	386	8	32½	8	....	6	3	No Stirrups.
	383	8	32½	12	....	6	3	" "
	384	8	32½	16	....	5	2	" "
	393	8	45	8	....	6	3	" "
	384	8	45	12	....	6	3	" "
	390	8	45	8	....	8	4	" "
	Effect of Distance from Support to First Stirrup.							
IV.	381	24	45	3	....	6	2	No Stirrups.
	396	24	45	3	8	6	2	Stirrups @ 8 in. Sp.
	397	24	45	3	12	6	2	" " " "
	398	24	45	3	16	6	2	" " " "
	388	14	45	0	....	4	1	No Stirrups.
	389	14	45	0	4	4	1	Stirrups @ 7 in. Sp.
	Miscellaneous.							
V.	380	....	....	....	....	None	....	No Web Reinf.
	399	12	45	12	....	5	2	
	400	12	22	0	....	4	1	

The behavior of the various beams under load was quite similar in many respects. At loads of 20 000 to 40 000 lb. tension cracks formed over the supports, extending to or across the longitudinal reinforcement. As the load was increased these cracks opened and extended and new cracks formed on either side of the support, converging diagonally toward the center of the support at the bottom of the

TABLE 6

PRINCIPAL RESULTS OF TESTS OF RESTRAINED BEAMS, SERIES OF 1917

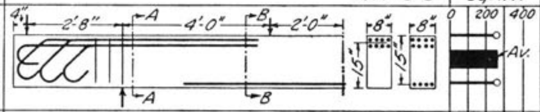
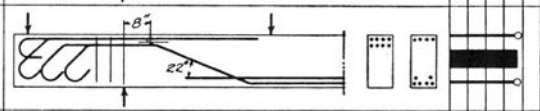
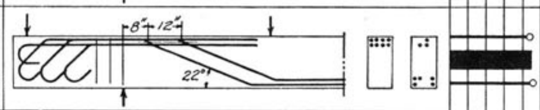
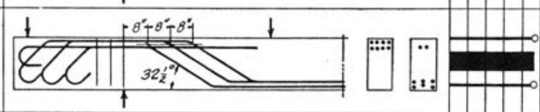
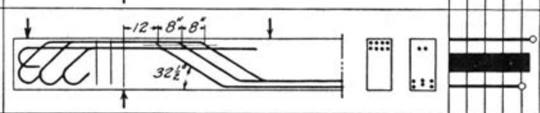
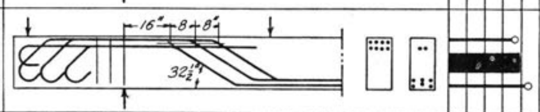
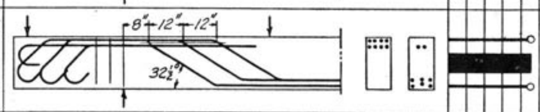
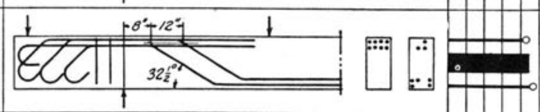
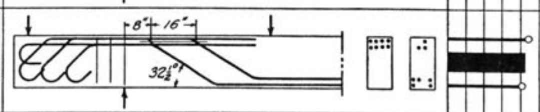
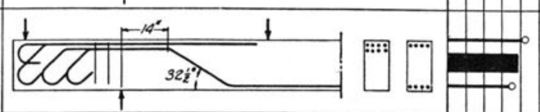
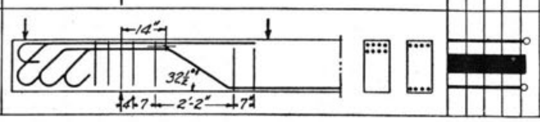
Beam No.	Description of Test Beams: Side Elevation; Sections A-A, B-B — Max. Shearing Stress at Support in lb. per sq. in.	Manner of Failure
380.1 380.2		Diagonal Tension. Diagonal Tension.
400.1 400.2		Diagonal Tension. Diagonal Tension.
382.1 382.2		Tension, followed by Diagonal Tension. Tension, Crushing at Bends & Diag. Tension.
386.1 386.2		Tension, Crushing at Bends. Tension, Crushing at Bends.
391.1 391.2		Tension. Tension & Diag. Tension.
392.1 392.2		Diagonal Tension. Tension & Diag. Tension.
383.1 383.2		Tension. Tension, Crushing at Bends & Diag. Tension.
385.1 385.2		Tension, Crushing at Bends. Tension at Supports and Center.
387.1 387.2		Tension, Crushing at Bends & Diag. Tension. Tension followed by Diagonal Tension.
388.1 388.2		Tension, Bond & Crushing at Bends. Bond & Diag. Tension.
389.1 389.2		Tension & Diag. Tension. Tension.

TABLE 6 (Continued)

PRINCIPAL RESULTS OF TESTS OF RESTRAINED BEAMS, SERIES OF 1917

Beam No.	Age at Test, Days	Reinforcement: Longitudinal, Per Cent: Support	Reinforcement: Longitudinal, Per Cent: Center	Reinforcement: Web: No. of $\frac{5}{8}$-in. ϕ, Bent Down	Reinforcement: Web: Spac., in.	Maximum Applied Load, lb.	Computed Stresses lb. per sq. in.: Longitudinal Steel: Support	Longitudinal Steel: Center	Shearing Stress: Support	Shearing Stress: Center	Bond: Support	Bond: Center	Cylinders Compress. Strength lb. per sq. in. Stored: In Sand	Stored: with Beam
380.1	63	1.95	0.93	None	—	102 800	27 400	26 000	258	229	137	242	2865	3060
380.2	59	1.95	0.93	None	—	104 000	28 000	26 300	261	231	138	245	3912	3665
Av.									260	230	137	243		
400.1	60	1.95	1.50	4 at 22°		151 000	40 000	26 600	376	370	196	252	3540	3158
400.2		1.95	1.50	4 at 22°		149 700	39 300	26 400	372	368	195	250	3098	3165
Av.									374	369	196	251		
382.1	62	1.96	1.27	5 at 22°	12	175 500	46 500	37 800	436	436	228	359	3305	3315
382.2	60	1.96	1.27	5 at 22°	12	183 700	48 100	39 500	456	456	238	375	3120	2748
Av.									446	446	233	367		
386.1	62	1.95	1.49	6 at $32\frac{1}{2}$°	8	188 200	50 100	33 100	465	460	246	316	3030	2870
386.2	60	1.95	1.49	6 at $32\frac{1}{2}$°	8	188 000	49 400	33 000	464	459	245	315	3470	3525
Av.									464	460	246	316		
391.1	60	1.93	1.54	6 at $32\frac{1}{2}$°	8	187 800	49 000	34 200	460	476	243	325	3185	2892
391.2	59	1.93	1.54	6 at $32\frac{1}{2}$°	8	172 000	45 500	31 300	422	437	223	298	3710	3495
Av.									441	456	233	212		
392.1	61	1.95	1.54	6 at $32\frac{1}{2}$°	8	146 400	39 200	27 300	363	371	193	258	3232	2818
392.2	61	1.95	1.54	6 at $32\frac{1}{2}$°	8	176 400	46 800	32 700	435	446	231	311	3320	2795
Av.									399	408	212	284		
383.1	63	1.95	1.43	6 at $32\frac{1}{2}$°	12	183 200	48 700	31 700	454	431	240	302	3172	3082
383.2	60	1.95	1.43	6 at $32\frac{1}{2}$°	12	181 500	47 900	31 500	450	428	237	300	2998	2950
Av.									452	430	238	301		
385.1	63	1.92	1.26	5 at $32\frac{1}{2}$°	12	176 300	46 400	37 900	430	433	229	361	3005	2985
385.2	59	1.92	1.26	5 at $32\frac{1}{2}$°	12	190 000	50 300	40 800	463	466	247	388	3710	3362
Av.									446	449	238	374		
387.1	62	1.91	1.23	4 at $32\frac{1}{2}$°	16	182 300	48 300	38 900	441	436	236	370	3268	3398
387.2	61	1.91	1.23	4 at $32\frac{1}{2}$°	16	168 400	43 900	36 100	408	403	218	342	3085	2965
Av.									424	420	227	356		
388.1	62	1.92	0.96	4 at $32\frac{1}{2}$°	—	173 800	45 600	43 100	425	401	225	409	3642	3260
388.2	60	1.92	0.96	4 at $32\frac{1}{2}$°	—	143 200	37 600	35 800	352	331	186	338	3202	2970
Av.									388	366	206	374		
389.1	62	1.96	0.96	4 at $32\frac{1}{2}$° $\frac{3}{8}$" ϕ stirrups.	—	174 000	46 300	43 400	434	400	227	412	3530	3210
389.2	61	1.96	0.96	4 at $32\frac{1}{2}$° $\frac{3}{8}$" ϕ stirrups.	—	169 000	44 600	42 200	422	388	221	400	3215	3102
Av.									428	394	224	406		

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TABLE 6 (Continued)

PRINCIPAL RESULTS OF TESTS OF RESTRAINED BEAMS, SERIES OF 1917

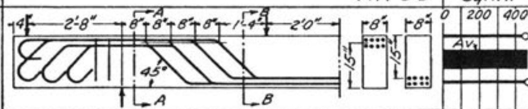
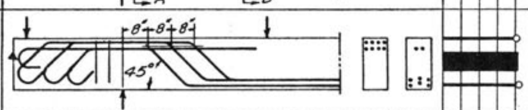
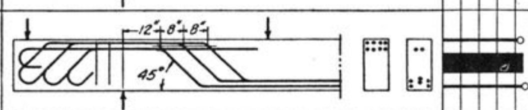
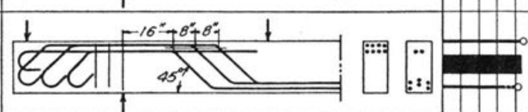
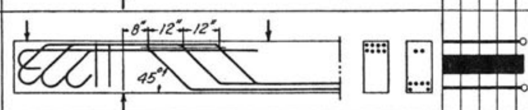
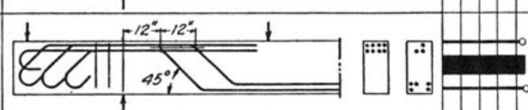
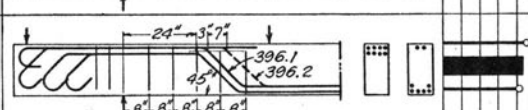
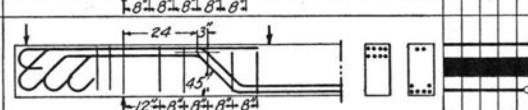
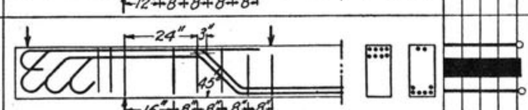
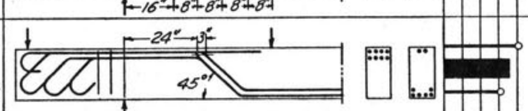
Beam No.	Description of Test Beams: Side Elevation; Sections A-A, B-B; Max. Shearing Stress at Support in lb. per sq. in. (0, 200, 400)	Manner of Failure
390.1 390.2		Tension, Crushing at Bends. Tension, Crushing at Bends.
393.1 393.2		Tension, Crushing at Bends. Tension & Diag. Tension.
394.1 394.2		Tension, Crushing at Bends. Tension, Crushing at Bends.
395.1 395.2		Tension, Crushing at Bends & Diag. Tension. Tension, Crushing at Bends & Diag. Tension.
384.1 384.2		Tension, Crushing at Bends & Diag. Tension. Tension, Crushing at Bends & Diag. Tension.
399.1 399.2		Tension, Crushing at Bends & Diag. Tension. Tension, Crushing at Bends.
396.1 396.2		Tension, Crushing at Bends (See Section of Text)
397.1 397.2		Tension, Crushing at Bends. Tension Crushing at Bends.
398.1 398.2		Diagonal Tension. Tension & Diag. Tension.
381.1 381.2		Crushing at Bends & Diagonal Tension. Diagonal Tension.

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TABLE 6 (Concluded)

PRINCIPAL RESULTS OF TESTS OF RESTRAINED BEAMS, SERIES OF 1917

Beam No.	Age at Test, Days	Reinforcement: Longitudinal, Per Cent: Support	Reinforcement: Longitudinal, Per Cent: Center	Reinforcement: Web: No. of $\frac{5}{8}$-in. φ, Bent Down	Reinforcement: Web: Spac., in.	Maximum Applied Load, lb.	Computed Stresses lb. per sq. in.: Longitudinal Steel: Support	Longitudinal Steel: Center	Shearing Stress: Support	Shearing Stress: Center	Bond: Support	Bond: Center	Cylinders Compress. Strength lb. per sq. in. Stored: In Sand	Stored: With Beam
390.1	62	1.99	2.04	8 at 45°	8	181200	48700	25400	458	466	238	242	2970	2905
390.2	58	1.99	2.04	8 at 45°	8	186000	49000	26000	469	477	244	248	3072	2735
Av.									464	472	241	245		
393.1	61	1.94	1.51	6 at 45°	8	164000	43700	30300	405	408	214	288	3150	3155
393.2	61	1.94	1.51	6 at 45°	8	170000	44700	31300	419	423	222	298	2878	2325
Av.									412	416	218	293		
394.1	59	1.95	1.55	6 at 45°	8	172300	44900	31600	427	442	220	301	3368	3145
394.2	60	1.95	1.55	6 at 45°	8	185400	49400	33900	458	475	241	324	3740	3355
Av.									442	458	230	312		
395.1	60	1.95	1.52	6 at 45°	8	180600	48000	33100	447	454	236	314	3112	3120
395.2	61	1.95	1.52	6 at 45°	8	167000	44100	30600	414	420	218	291	3310	3015
Av.									430	437	227	302		
384.1	63	1.97	1.46	6 at 45°	12	176700	47300	24200	442	431	233	230	3210	3080
384.2	59	1.97	1.46	6 at 45°	12	178600	47800	24500	447	436	236	233	3638	3442
Av.									444	434	234	232		
399.1	60	1.95	1.23	5 at 45°	12	176100	47000	37600	435	423	231	357	3335	3352
399.2	60	1.95	1.23	5 at 45°	12	184900	49000	39300	456	443	242	374	3035	2810
Av.									446	433	236	366		
396.1	59	1.95	1.46	6 at 45° $\frac{3}{8}$" φ stirrups	3	182600	48100	32000	453	438	238	305	3485	3410
396.2	60	1.95	1.46	6 at 45° $\frac{3}{8}$" φ stirrups	10	165000	43700	29000	410	396	216	276	3715	3362
Av.									432	417	227	290		
397.1	63	1.98	1.50	6 at 45° $\frac{3}{8}$" φ stirrups	3	192500	51900	34500	484	475	253	331	3128	3235
397.2	61	1.98	1.50	6 at 45° $\frac{3}{8}$" φ stirrups	3	179600	47500	32300	451	444	236	309	3045	2682
Av.									468	460	244	320		
398.1	60	1.93	1.48	6 at 45° $\frac{3}{8}$" φ stirrups	3	168000	44500	30300	412	407	218	288	2860	2682
398.2	60	1.93	1.48	6 at 45° $\frac{3}{8}$" φ stirrups	3	173100	45200	31200	424	419	225	297	3412	2990
Av.									418	413	222	292		
381.1	61	1.94	1.45	6 at 45°	3	165000	44100	29200	408	393	218	277	3250	3070
381.2	61	1.94	1.45	6 at 45°	3	124100	33200	22200	309	296	165	209	3692	3385
Av.									358	344	192	243		

6-2-2

beam. Since the amount of longitudinal steel at mid-span was generally 50 per cent greater in proportion to the bending moment than at the support, tension cracks did not usually appear at mid-span until a load of about 80 000 lb. was applied. With further increase in load new cracks formed near the support, radiating in a fan-like pattern about the center of bearing at the support. It is difficult to specify where these cracks cease to be tension cracks and become diagonal tension cracks.

Detailed notes of the tests, giving specific phenomena of each test, are presented here to aid in furnishing a conception of the manner of failure of these beams.

Beams 380.1 and 380.2. In these beams the eight bars over the support and the five bars at the bottom of the beam were extended past the section of contraflexure a distance of 20 diameters. In beam 380.1 a diagonal tension crack opened rapidly near the south support at a load of 98 000 lb. and the beam failed suddenly at this crack when the load reached 102 800 lb. The crack extended diagonally upward to the longitudinal bars, then followed along the bars toward the inner load point, stripping off the concrete above the bars at failure. At a load of 96 000 lb. a slight negative slip, or movement due to compressive stress, was measured at the ends of longitudinal bars near the inner load point.

In beam 380.2 a diagonal crack was first noted, just inside the south support, at a load of 61 400 lb. Similar cracks formed at the north support at a load of 81 800 lb. Failure occurred suddenly through the opening of the diagonal crack at the south support at a load of 104 000 lb. and was similar to the failure of beam 380.1. Figure 10 shows a view of the portion of the beam in which failure occurred.

Beams 381.1 and 381.2. These beams had six reinforcing bars bent down in two layers 3 in. apart, so that all intersected the section of contraflexure at about mid-depth of the beams. In beam 381.1, diagonal cracks were noted at a load of 70 000 lb., but failure did not occur until a load of 165 000 lb. was applied, when the concrete began to crush under the upper bends in the bars inside the north support and a sudden violent diagonal tension failure quickly followed.

In beam 381.2 diagonal cracks about 0.01 in. wide were noted at a load of 83 000 lb. at both ends of the beam extending from the upper bends in the bars toward the supports. At a load of 105 500 lb. the diagonal crack at the north end of the beam was 0.03 in. wide. As further load was applied the diagonal cracks opened slowly until

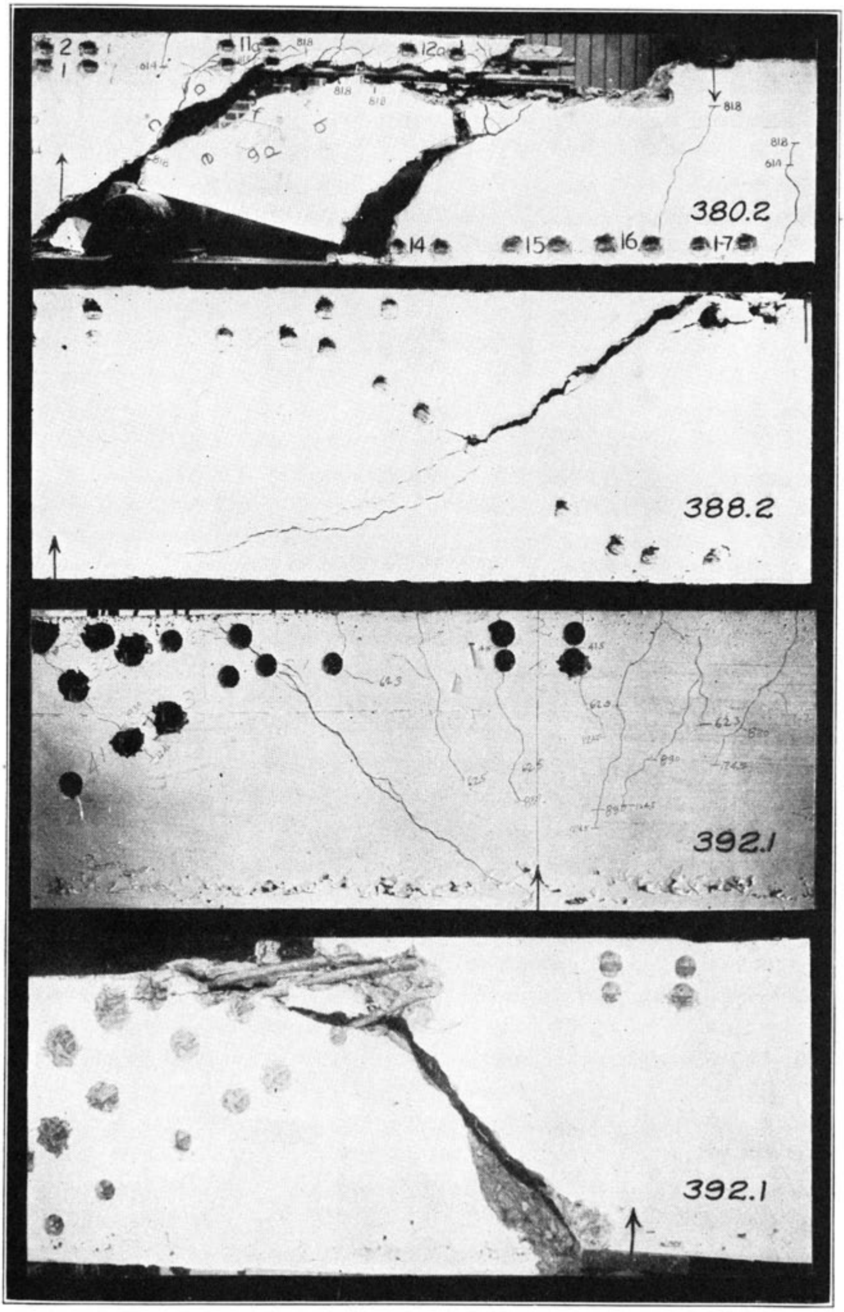

FIG. 10. VIEW OF PORTION OF BEAMS 380.2, 388.2, AND 392.1 AFTER TEST

at a load of 120 000 lb. the diagonal crack at the north end began to open rapidly. Failure occurred at a load of 124 100 lb.

It is evident that in these beams the inclined bars were located too far from the support to prevent diagonal tension failures.

Beams 382.1 and 382.2. Diagonal cracks were first noted in beam 382.1 near the supports at a load of 81 600 lb.; at a load of 142 800 lb. a diagonal crack was noted near the north point of contraflexure, and at a load of 163 000 lb. a similar crack formed near the south point of contraflexure. At a load of 166 000 lb. the yield point of the steel over the support was exceeded, and the width of cracks near the support had increased to 0.03 in. The beam took further load up to 175 500 lb., when rapid yielding began and a secondary diagonal tension failure occurred.

The formation of diagonal cracks in beam 382.2 was similar to that in beam 382.1; cracks were first noted at a load of 83 000 lb. and had reached a width of 0.01 in. at a load of 169 500 lb. Crushing under the upper bends of inclined bars was first noted at a 175 500-lb. load; tension cracks at this load were 0.18 wide over supports, indicating tension failure. At the maximum load of 183 700 lb. complete failure occurred by the opening of a diagonal crack.

Beams 383.1 and 383.2. No unusual action of beam 383.1 occurred up to a load of 166 200 lb., when crushing began under the upper bends of inclined bars. The beam failed at a load of 183 200 lb. by tension in the steel at the south support. The diagonal cracks that formed remained small.

In beam 383.2 at a load of 168 000 lb. a diagonal crack had opened about 0.06 in. at the first bend from the north support, and the bent bar had pulled away slightly from the concrete above. With a small increase in load a similar cracking and crushing developed at the south end. At a load of 181 500 lb. the beam failed at a diagonal crack extending upward from the support at about 45 deg. to the horizontal. The failure was not due primarily to diagonal tension, since the crushing of concrete under bends destroyed much of the effectiveness of the bars in resisting web stresses.

Beams 384.1 and 384.2. No diagonal cracks of large size formed in beam 384.1. At a load of 164 400 lb. the yield point of the steel over supports was exceeded and slight crushing was noted under bends of bars. Slip measurements on top bars ending near the load points showed negative slip, or movement due to compressive stress in these bars. Yielding of the longitudinal steel over the support resulted in widespread cracking, and complete failure occurred at an inclined crack near the south support at a load of 176 700 lb.

In beam 384.2 the yield point was reached on the longitudinal reinforcement at a load of 159 400 lb., and at a load of 169 600 lb. tension cracks were 0.12 in. wide at the north support. Soon thereafter a diagonal crack 0.01 in. wide formed near the south support. Crushing under bends of bars took place as the load was increased to a maximum of 178 600 lb. The yield point of the first bent-down bar at the south end of the beam was reached at failure, which was evidently caused by the crushing of the concrete under bends.

Beams 385.1 and 385.2. In beam 385.1 diagonal cracks were first noted at a load of 83 500 lb., developing to a width of 0.02 in. at a load of 165 000 lb., when the yield point of the longitudinal steel over the supports was exceeded. Slight slip occurred in top bars near the inner load point at the north end, but not at the south end. Failure by tension came at the maximum load of 176 300 lb.

In beam 385.2 inclined cracks opened near the supports and near inner load points. At a load of 168 000 lb. the latter had extended well across the section of contraflexure, and, as the load increased, tension failure occurred at both supports and at the middle of the beam. At a load of 190 000 lb. a secondary compression failure occurred near the north inner load point.

Beams 386.1 and 386.2. The first inclined cracks formed in beam 386.1 about 8 in. inside the support and with increasing loads other cracks formed at flatter angles of inclination. At loads of 164 800 to 174 000 lb. diagonal cracks formed just outside of and extending toward the inner load points. Diagonal cracks near the supports opened up as much as ⅛ in. at the maximum load, although failure was due to yielding of the longitudinal steel at the supports and crushing of concrete at the first bend inside the south support.

The behavior of beam 386.2 was similar to that of beam 386.1. At the maximum load of 188 000 lb. crushing of the concrete occurred under the first bend inside the south support, and the steel over the supports had passed its yield point.

Beams 387.1 and 387.2. At a load of 165 000 lb. on beam 387.1 tension cracks were increasing over the north support. Crushing of concrete was noted under bends at a load of 176 000 lb. With increase in load, diagonal cracks formed in the region of the section of contraflexure, and crushing and spalling of concrete under bends began. After the maximum load of 182 300 lb. the beam yielded slowly until failure occurred suddenly by the opening of a diagonal crack near the north support.

In beam 387.2 the usual diagonal cracks were observed near the supports at a load of 62 600 lb., and near the section of contraflexure at a load of 125 300 lb. At a load of 157 700 lb. the longitudinal steel had passed its yield point, as had the first inclined bar from the south support; the diagonal cracks began to open and failure occurred gradually at a load of 168 400 lb. at a diagonal crack near the south support.

Beams 388.1 and 388.2. Diagonal cracks were first noted near supports at a load of 62 800 lb. on beam 388.1, and at a 125 500-lb. load they had extended to intersect the inclined bars at about mid-height of the beam. At a 172 000-lb. load tension cracks near the north support and the north load point were about 0.1 in. wide, indicating that the yield point had been reached in the longitudinal reinforcement. At the maximum load of 173 800 lb. slipping of the straight top bars near the inner load points was observed, and as this slip increased to about one inch the load dropped off and the concrete under the bent bars crushed and spalled badly.

Beam 388.2 behaved in a manner similar to beam 388.1, with respect to development of diagonal cracks. At a load of 128 000 lb. there was evidence of slipping of the straight top bars near the south inner load point, and diagonal cracks near the south section of contra-flexure were 0.3 in. wide. At a 143 200-lb. load a large diagonal crack across the inclined bars opened due to slip of straight bars and to yielding of the inclined bars. As the failure progressed the straight top bars stripped loose and the concrete under bends crushed slightly. Figure 10 shows the portion of the beam in which failure occurred.

Beams 389.1 and 389.2. These beams were similar to beams 388.1 and 388.2 except that four vertical stirrups were added between support and inner load point. Diagonal cracks developed about as in the preceding beams. At a load of 174 000 lb. the tension crack over the south support was 0.12 in. wide, and the diagonal crack crossing the inclined bars at about mid-height of the beam was 0.18 in. wide, indicating that the yield point of the steel had been reached at both places. As the diagonal crack opened further the load fell off and sudden failure occurred along the line of the diagonal crack.

In beam 389.2 diagonal cracks were noted at a load of 102 000 lb. At a load of 166 000 lb. tension cracks over the support were 0.12 in. wide, a diagonal crack crossing the stirrups near the south support was 0.05 in. wide, and a large tension crack opened near mid-span. A secondary failure by crushing of concrete at mid-span occurred at a load of 169 000 lb.

Beams 390.1 and 390.2. In these beams the eight bars over the support were bent down at four different points and were all used in the region of positive moment.

In beam 390.1 the yield point of the steel over the supports had been reached at a load of 170 000 lb.; diagonal cracks near the supports were 0.03 to 0.12 in. wide, and concrete under bends had begun to crush. The load increased slowly to a maximum of 181 200 lb. After the maximum load the concrete split off at the side of the beam outside the support.

In beam 390.2 the typical diagonal cracks appeared at loads of 60 700 to 163 500 lb. At the latter load a tension crack at the north support was 0.04 in. wide, and concrete had crushed slightly under bends near the south support. At a load of 175 000 lb. the tension crack at the north support was 0.18 in. wide, and with a slight increase in load a large diagonal crack opened and crushing occurred under bends near the south support. After the maximum load of 186 000 lb. the concrete split and shattered off outside the south support as in the preceding test.

Beams 391.1 and 391.2. Diagonal cracks were first noted in beam 391.1 at a load of 42 400 lb., and at a load of 163 400 lb. these cracks were quite generally distributed between inner load points and supports. At a load of 167 500 lb. there was evidence of slipping of horizontal bars near the north support at first and second bends. Cracks near the support were 0.03 to 0.05 in. wide, and at 184 000 lb. the tension cracks over supports were 0.12 to 0.25 in. wide. At the maximum load of 187 800 lb. failure occurred by opening of a diagonal crack outside the south support, accompanied by crushing of the concrete under bends.

In beam 391.2 the diagonal cracks were small until a load of 164 400 lb. was reached, when the yield point of the longitudinal steel was passed near the north support. The load was increased slowly to 172 000 lb., when failure occurred suddenly along a diagonal crack near the north support. The inclined bars were not in a position to prevent the opening of this crack.

Beams 392.1 and 392.2. These beams were like beams 386 and 391 except that the inclined bars were located farther from the supports and were evidently not in position to prevent the formation of diagonal cracks near the supports.

In beam 392.1 diagonal cracks formed at loads of 83 000 to 124 500 lb., attaining a width of 0.05 in. on a crack extending at about 45 deg. to the horizontal toward the south support, as seen in Fig. 10. Failure occurred by diagonal tension along this crack at a load of 146 400 lb., no crushing or large tension cracks having developed. A view of the beam after failure is also shown in Fig. 10.

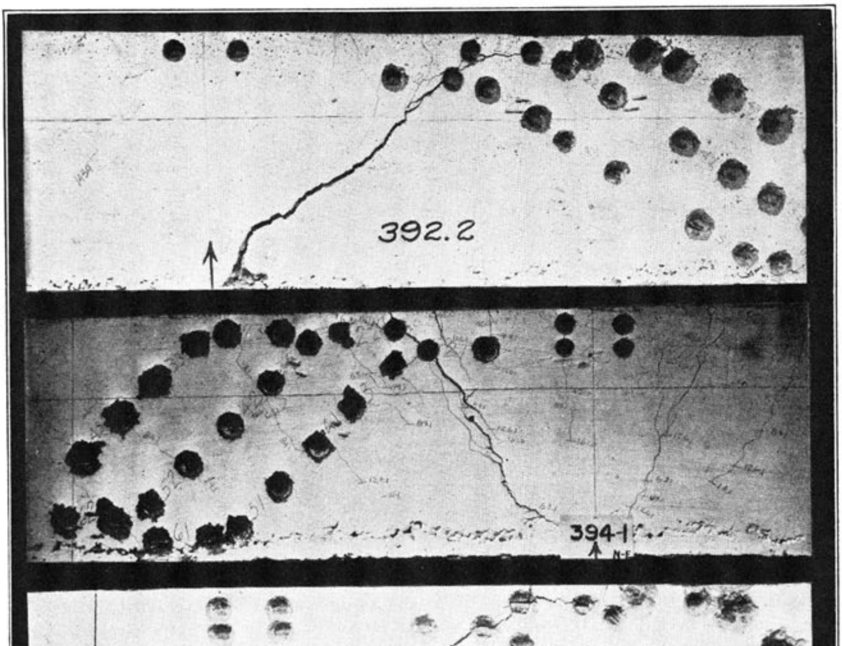

FIG. 11. VIEW OF PORTION OF BEAMS 392.2, 394.1, 395.1, AND 399.2 AFTER TEST

At a load of 123 300 lb. diagonal cracks near the supports in beam 392.2 were 0.01 and 0.02 in. wide at the level of the longitudinal bars. At a 175 300-lb. load the crack near the north support was 0.18 in. wide, and concrete spalled slightly under the bend where it was intersected by cracks. The beam failed suddenly along diagonal cracks at both ends near the supports at a load of 176 400 lb. Figure 11 indicates that the diagonal crack at failure did not intersect the inclined portions of the reinforcing bars.

Beams 393.1 and 393.2. Diagonal cracks appeared in beam 393.1 at loads of 62 000 to 164 000 lb. when a large tension crack opened near the first bend inside the north support, and crushing and spalling of concrete under the first and second bends followed.

The behavior of beam 393.2 was similar to that of beam 393.1 up to the opening of a tension crack 0.04 in. wide at the first bend near the north support at a load of 170 000 lb. Crushing of concrete under bends was noted at this load, and large diagonal cracks opened at both ends of the beam near the section of contraflexure. Failure occurred at the load of 170 000 lb. along a diagonal crack crossing the lower bends of inclined bars and extending to the north inner load point.

Beams 394.1 and 394.2. At a load of 63 100 lb. on beam 394.1 the usual inclined cracks appeared near the supports, intersecting the longitudinal bars at the points where they were bent down. As the load increased diagonal cracks formed near the section of contraflexure. At a load of 161 000 lb. diagonal cracks extending from the first bend toward the support were about 0.06 in. wide, indicating that the longitudinal bars had reached the yield point. At the maximum load of 172 300 lb. the tension crack over the south support was 0.12 in. wide and the concrete crushed under the first bend at the north end of the beam. A view of the beam after failure is given in Fig. 11.

Diagonal cracks formed in beam 394.2 in a manner similar to those of beam 394.1. At a load of 163 000 lb. the diagonal crack at the first bend inside the south support was 0.08 in. wide, indicating yielding of the reinforcement. Crushing under the first bend at each end of the beam began at a 175 000-lb. load, and continued until the maximum load of 185 400 lb. was reached.

Beams 395.1 and 395.2. These beams were like beams 393 and 394 except that the inclined bars were placed farther from the supports. Like beams 394.1 and 394.2, they failed primarily by tension and finally along diagonal cracks that did not intersect the inclined portion of the bent-down bars. In beam 395.1 the yield point of the steel over supports was passed at a load of 150 000 lb., and at a load of 165 000 lb. the concrete began to crush under bends in reinforcing bars. Yielding and crushing increased as loading progressed until sudden failure occurred at a diagonal crack near the south support. A view of the beam after failure is given in Fig. 11.

In beam 395.2 diagonal cracking began at a load of 61 400 lb. at the first bend inside the support; at a load of 160 000 lb. the width of the cracks (0.04 in.) indicated yielding of the reinforcement. Failure occurred at a load of 167 000 lb., when extensive spalling took place under the bends at the south end of the beam.

Beams 396.1 and 396.2. These beams had six bars bent down through the section of contraflexure, and also had vertical stirrups spaced 8 in. apart, beginning 8 in. from the supports. Tension cracks opened considerably over the supports in beam 396.1 at a load of 123 600 lb., and at a load of 164 800 lb. the steel over the south support had passed the yield point. Slight spalling of concrete under some of the bends occurred as the load was increased to a maximum of 182 600 lb.

Two of the six bent-down bars in beam 396.2 were placed by mistake 10 in. further from the support than were the corresponding bars in beam 396.1; however, the behavior of the two beams was similar. The yield point of the steel over the supports in beam 396.2 was reached at a load of 165 000 lb., and failure occurred at this load due to the shearing off of concrete outside the hooks at the north outer load point.

Beams 397.1 and 397.2. In beam 397.1 the typical vertical and diagonal cracks formed near the supports, the cracks following the plane of the vertical stirrups in some cases. The yield point of the steel at supports was reached at a load of 168 000 lb., and as further load was applied the diagonal crack near the south support opened to a width of 0.12 in. The maximum load of 192 500 lb. was reached when the concrete crushed under the bends at the south end of the beam.

In beam 397.2 the first diagonal cracks across stirrups were noted at a load of 82 200 lb. At a load of 169 000 lb. a tension crack over the south support had opened about 0.10 in. Strain readings taken on the stirrup nearest the south support at a load of 172 000 lb. showed that the stirrup steel had passed the yield point. The maximum load of 179 600 lb. was reached when the concrete under bends at the north end of the beam spalled considerably.

Beams 398.1 and 398.2. In these beams the first stirrup was located 16 in. inside the support, and was not very effective in preventing diagonal tension failure. Diagonal cracks formed in beam

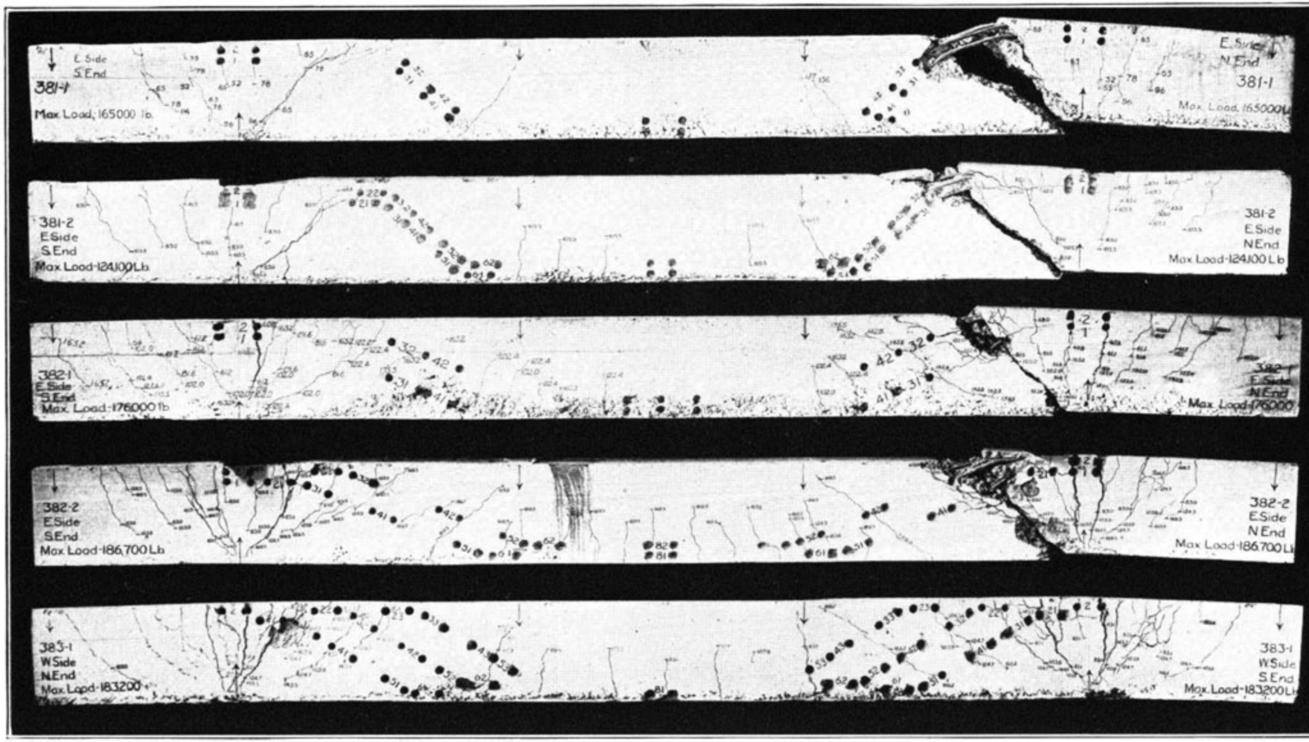

Fig. 12. Views of Beams after Testing, Series of 1917

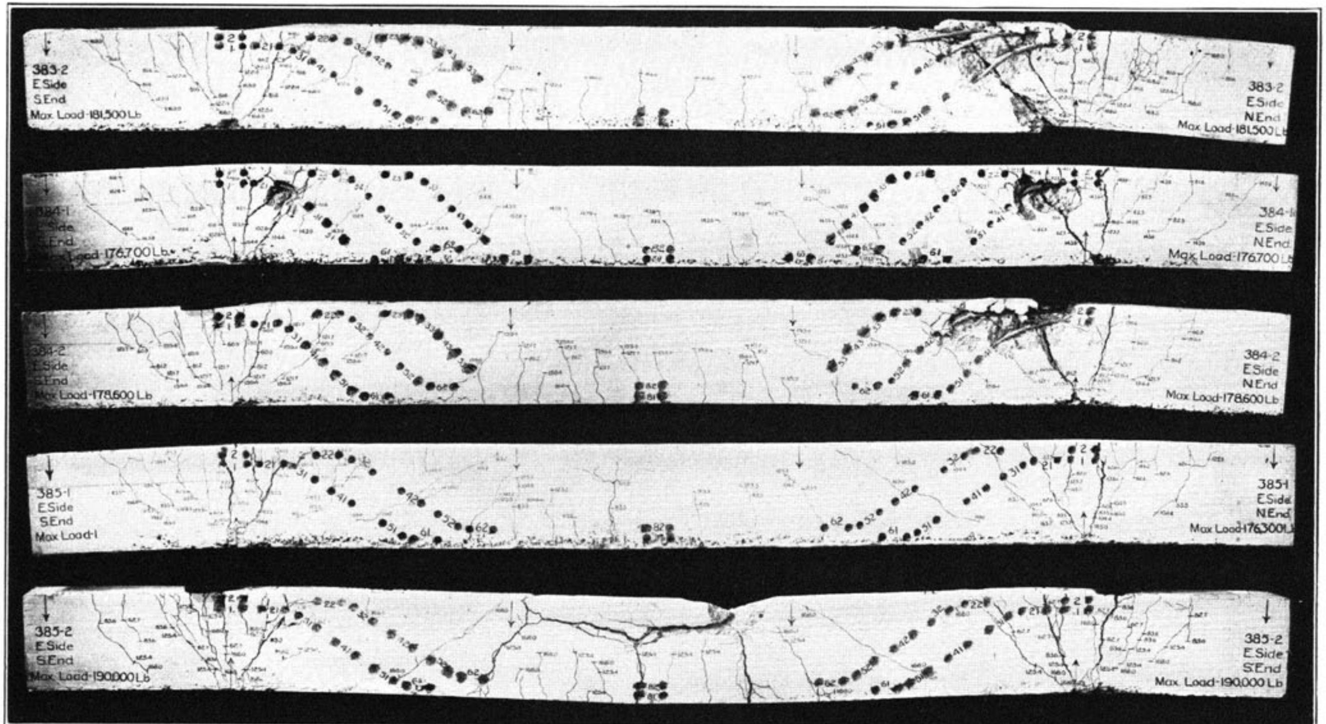

FIG. 13. VIEWS OF BEAMS AFTER TESTING, SERIES OF 1917

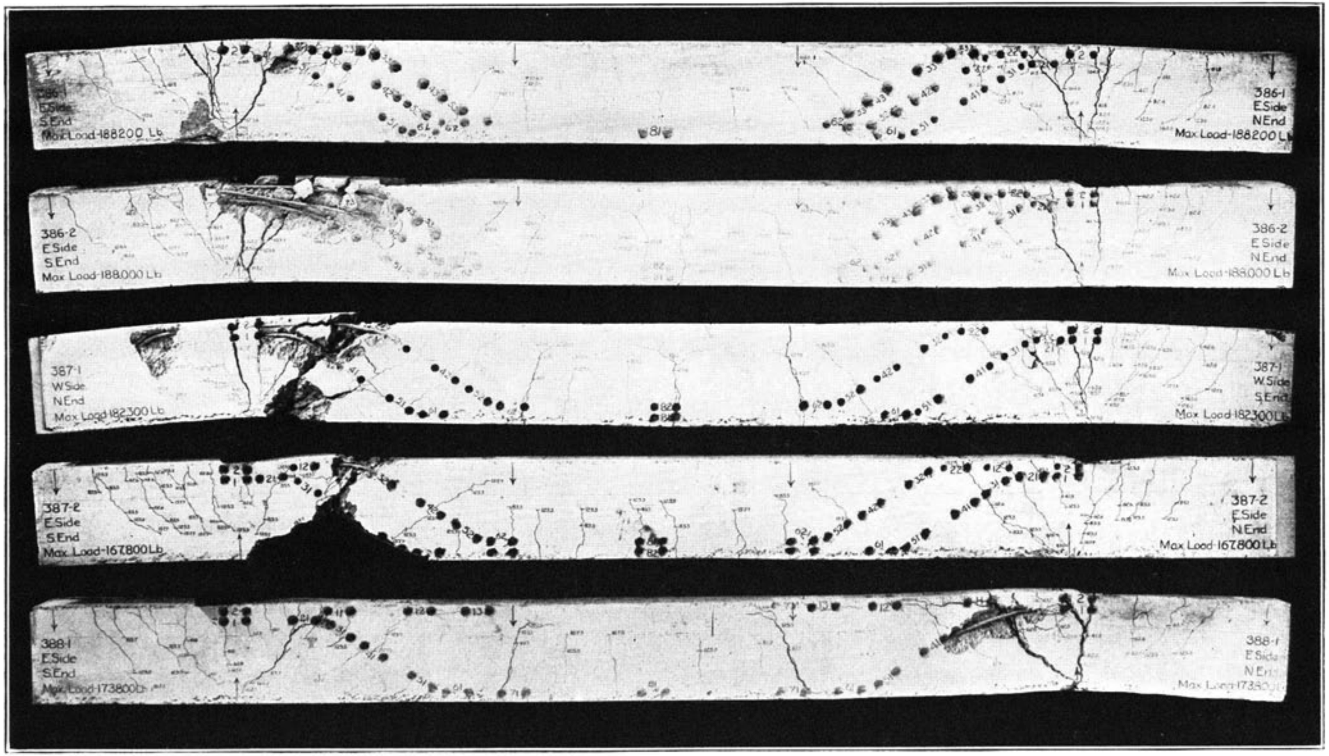

FIG. 14. VIEWS OF BEAMS AFTER TESTING, SERIES OF 1917

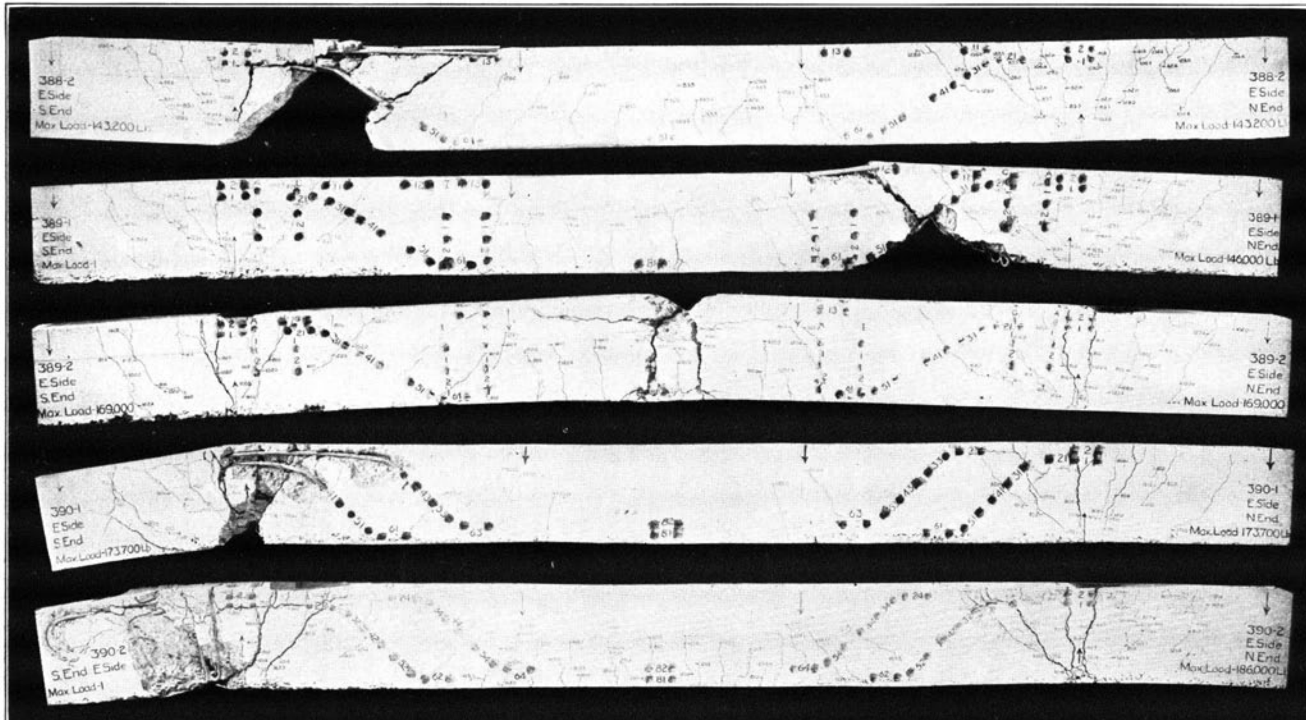

Fig. 15. Views of Beams after Testing, Series of 1917

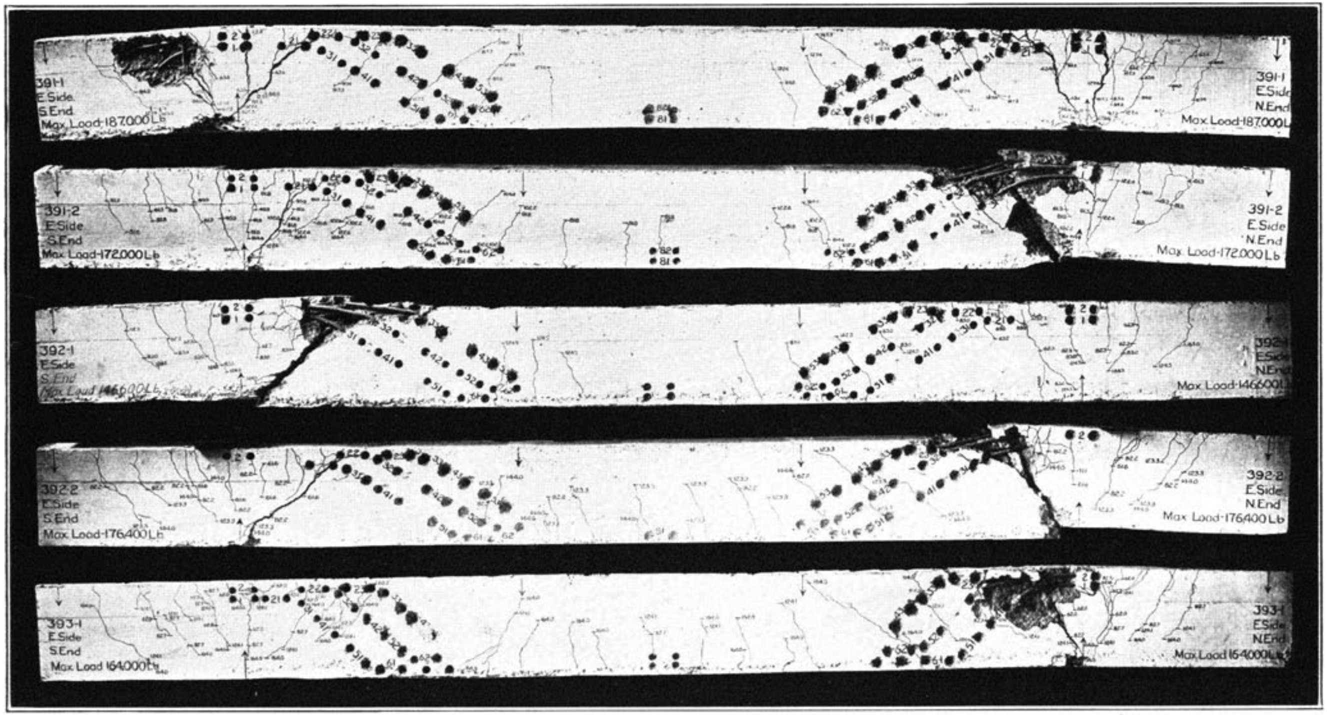

FIG. 16. VIEWS OF BEAMS AFTER TESTING, SERIES OF 1917

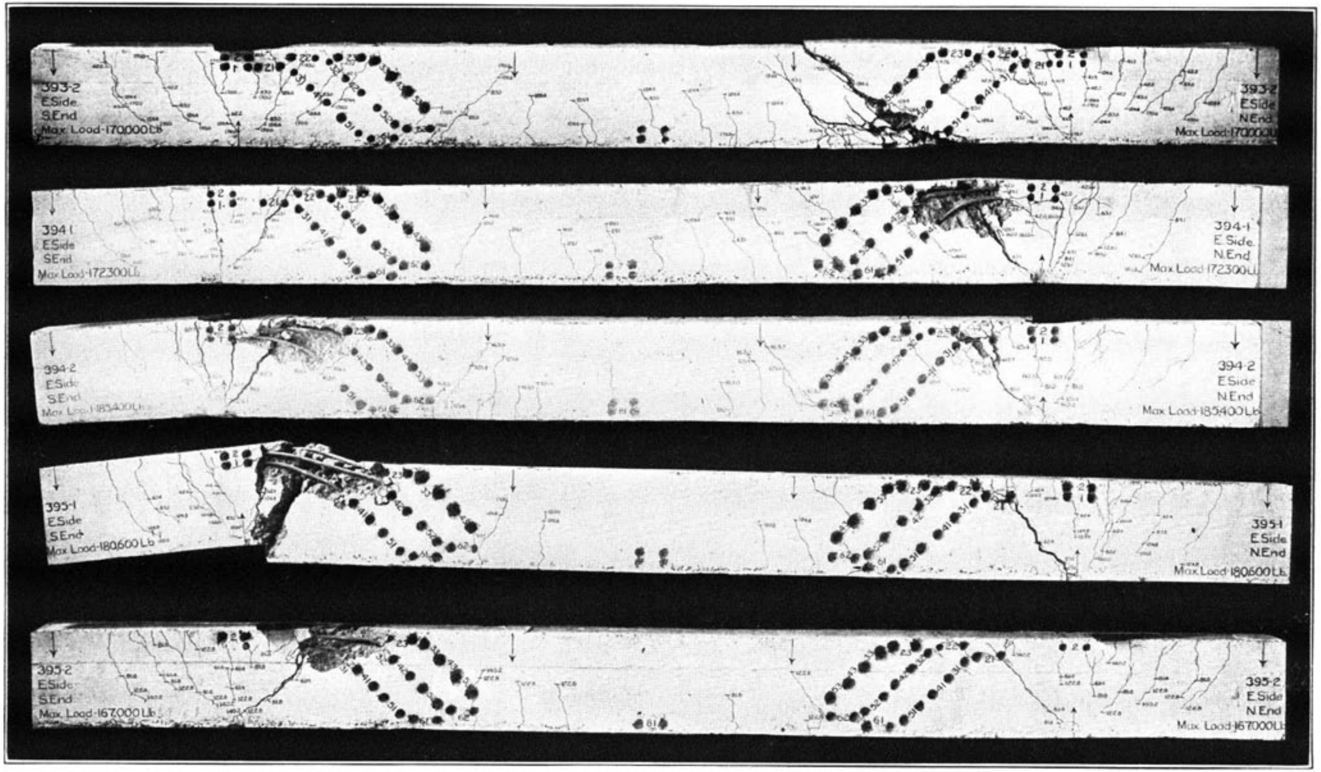

Fig. 17. Views of Beams after Testing, Series of 1917

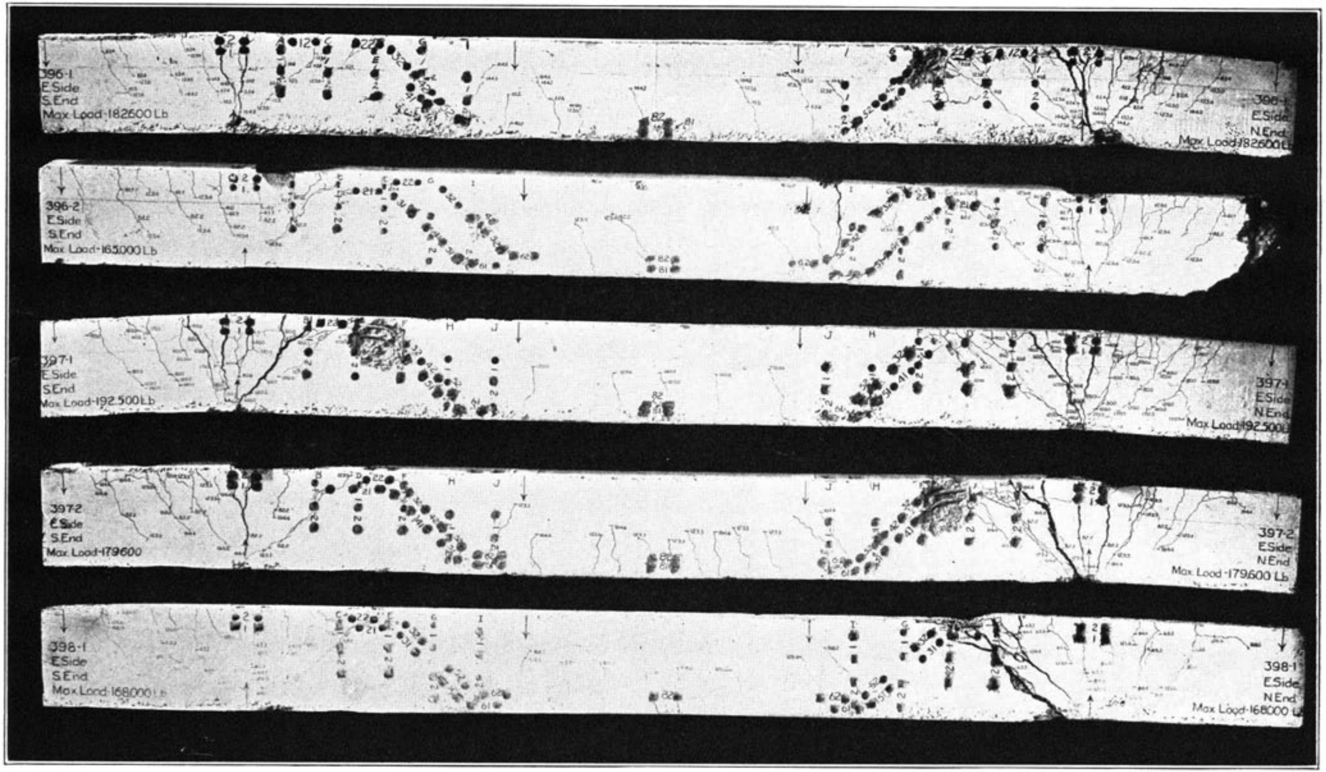

FIG. 18. VIEWS OF BEAMS AFTER TESTING, SERIES OF 1917

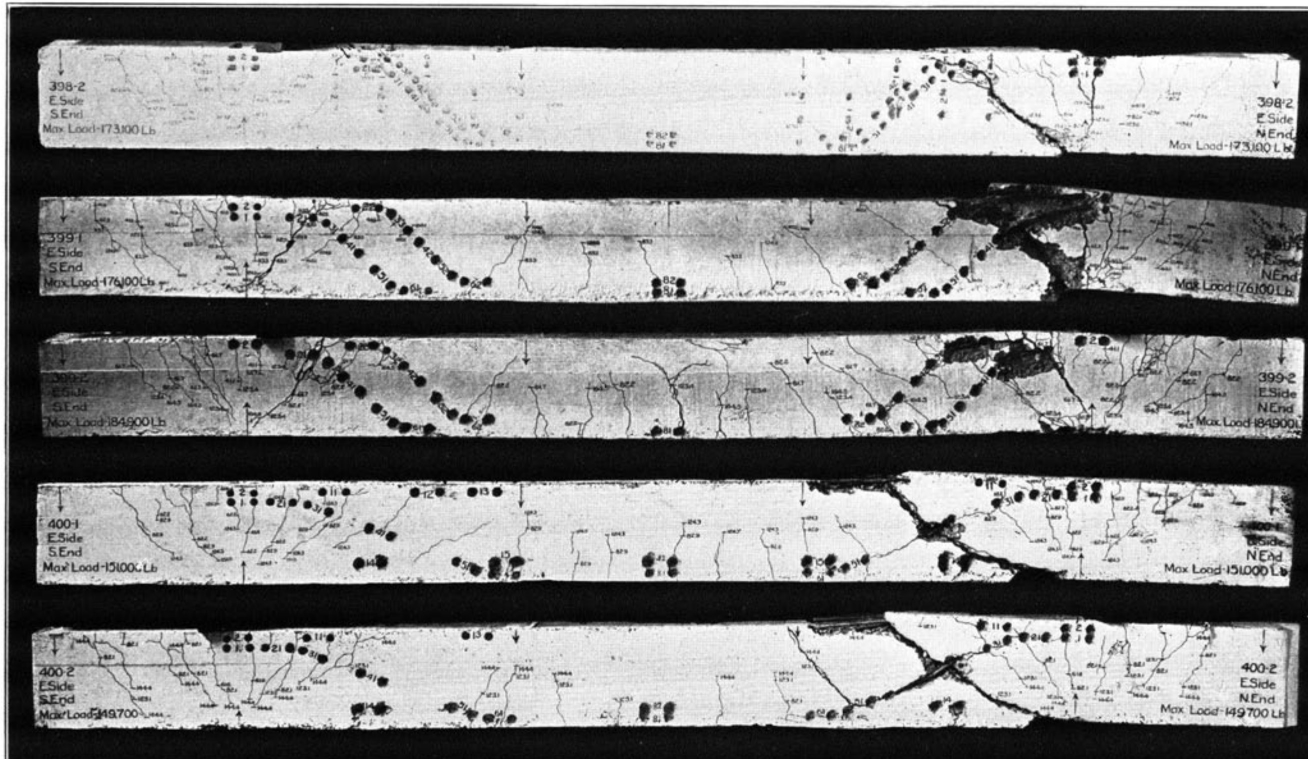

Fig. 19. Views of Beams after Testing, Series of 1917

398.1 between the support and the first stirrup, beginning at a load of 63 300 lb. At a load of 126 600 lb. these cracks were 0.01 to 0.03 in. wide. At a load of 168 000 lb. failure occurred suddenly at the diagonal crack near the north support.

Near each support of beam 398.2 a diagonal crack formed across the first stirrup and extended toward the support at a load of 82 400 lb. At a load of 146 000 lb. strain measurements showed that the first stirrup inside the north support was stressed to the yield point. At a load of 159 700 lb. the steel over the north support reached its yield point, and a large tension crack opened. The beam failed along a diagonal crack near the north support at a load of 173 100 lb.

Beams 399.1 and 399.2. At a load of 83 300 lb. on beam 399.1, typical diagonal cracks had formed across the first bend inside both supports, and at a load of 124 900 lb. these cracks were about 0.01 in. wide. The diagonal crack near the north support had become 0.05 in. wide at a load of 160 100 lb., and the concrete was beginning to crush under bends of the reinforcement. At the maximum load of 176 100 lb. the concrete spalled at bends at the north end of the beam and sudden failure occurred along a diagonal crack.

In beam 399.2 at a load of 164 500 lb. the yield point of the longitudinal bars near the first bend was reached. With further loading the concrete began to crush at the first bend at the north end, and a diagonal crack near the north support opened to a width of 0.20 in. At the maximum load of 184 900 lb. the concrete spalled under bends at the north end of the beam (see Fig. 11), causing a decrease in the load.

Beams 400.1 and 400.2. The web reinforcement in these beams consisted of four bars bent down in one layer at an angle of 22 deg. to the horizontal. Typical diagonal cracks formed in beam 400.1 near the support and were followed at the load of 124 300 lb. by diagonal cracks that crossed the inclined bars at about mid-height of the beam. Failure occurred by the rapid opening of one of these cracks at the north end of the beam at a load of 151 000 lb.

The action of beam 400.2 was similar to that of beam 400.1. The diagonal crack at which failure occurred crossed inclined bars about midway between the north support and the inner load point. This crack was first noted at a load of 123 100 lb. and opened rapidly when the yield point of the steel was reached at a load of 144 400 lb. Failure occurred suddenly at a load of 149 700 lb.

Views of all the beams after test are given in Figs. 12 to 19. These views show the location of the gage lines and of the cracks with reference to the load points and supports. The indications of final failure shown in the figures must not be confused with the initial cause of failure, which was generally much less evident. These illustrations will be found most useful when examined in connection with the detailed notes of the tests.

17. *Résumé of Characteristics of Failure.*—Reference to Section 16 and to Table 6 shows that failure in but few of the beams was strictly due to diagonal tension. In the majority of cases the primary failure occurred when the yield point of the longitudinal steel over the support or near bends was reached; this yielding of the steel produced numerous tension cracks, which developed into diagonal cracks as the loading proceeded. Local slipping of longitudinal steel over the support, while not generally present, had much the same effect as yielding of the steel. In beam 388.2 this slipping evidently caused premature failure of the beam. The crushing of the concrete inside of bends, which were all of 3-inch radius (4.8 diameters of bar), contributed in many cases to a gradual yielding and final failure of the beam.

It will be noted that only seven beams are listed in Table 6 as cases of primary failure by diagonal tension. Of these, beams 380.1 and 380.2 had no web reinforcement, and did not develop high tension or bond stresses. The shearing unit stress in these beams at failure averaged 260 lb. per sq. in. Beam 381.2, in which the first bent-down bar was 24 in. from the support, failed by a diagonal tension crack which did not intersect any of the web reinforcement. The shearing unit stress on this beam, which may be considered as having no web reinforcement in the region of failure, was 309 lb. per sq. in., and its companion, which failed initially in tension, developed a shearing unit stress of 408 lb. per sq. in. Beam 392.1, in which the first bend was 16 in. from the support, also failed at a diagonal crack crossing the region between the support and the first inclined bar and intersecting the reinforcement at the first bend. The maximum nominal shearing unit stress on this beam was 363 lb. per sq. in.; the companion beam developed 435 lb. per sq. in. before it failed by tension and diagonal tension. Beam 398.1 had its first bend located 24 in. from the support, and its first stirrup 16 in. from the support. The diagonal cracks causing failure intersected the top gage line of the first stirrup and evidently stressed the stirrup beyond the yield point; other large cracks formed near the support where there was no web reinforcement. This beam and beams 400.1 and 400.2 may be said to be the only beams failing by diagonal tension in which the web reinforcement was in a position to resist the failure. In the latter two beams the bent bars were inclined 22 deg. to the horizontal and were all in

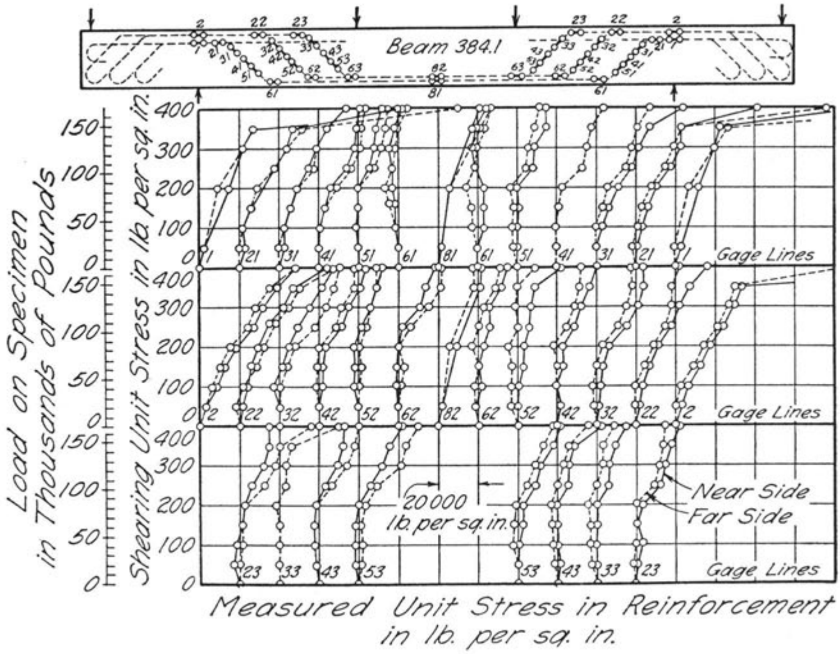

FIG. 20. LOAD-STRESS CURVES FOR BEAM 384.1

one layer. The diagonal cracks causing failure intersected the inclined bars squarely at mid-depth of beam, but the inclination of the bars was evidently too small to make them very effective in resisting diagonal tension.

It is interesting to note that the shearing stresses in beams that failed by diagonal tension ranged from 258 to 309 lb. per sq. in. where there was no web reinforcement at the section of failure, and from 363 to 412 lb. per sq. in. when the diagonal crack intersected the web members to some extent; the corresponding stress in the one beam failing primarily by bond was 352 lb. per sq. in., while in beams failing by tension or by crushing of concrete at bends the shearing stress ranged from 405 to 484 lb. per sq. in.

18. *Measured and Calculated Stresses in Web Reinforcement.*—Since few of the beams of the series failed in the web, the principal information concerning their web resistance will be found in a comparison of measured and calculated stresses in the web reinforcement. From the strain measurements taken on both longitudinal and inclined reinforcement, load-stress curves have been plotted for all of the beams tested. Figure 20 shows load-stress curves for beam 384.1,

which may be considered as typical for the series. The curves show the variation in stress with load in various parts of the beam. These load-stress curves show clearly where high stresses were developed; for example, it is seen that at a load producing a shearing stress of 350 lb. per sq. in. the stresses at several gage lines were near the yield point of the reinforcement; as the shearing stress was increased to 400 lb. per sq. in. the yield point was reached at gage lines 1, 2, and 21, which were on horizontal bars over or near the support.

One of the main objects of this investigation was to compare the measured stresses with the stresses calculated by use of the equations of Section 3. Figure 21 shows load-stress curves for certain gage lines on the inclined bars of most of the beams of the series. Each curve represents the average of stress measurements at corresponding gage lines at each side and each end of two companion beams. The location of the various gage lines is shown in Figs. 12 to 19. Gage lines 31 and 41 were the upper and middle gage lines, respectively, on the first inclined bar inside the support, gage lines 32 and 42 are similarly located on the second inclined bar, and so on. The greatest stress was generally observed on gage line 31, though in a few cases it was found on gage line 32. All of the curves of Fig. 21 have one characteristic in common; the measured stress in the steel was quite small until a shearing stress of 150 to 200 lb. per sq. in. was reached, beyond this the increase in stress with load was quite rapid.

A comparison may be made between the measured stresses on gage line 31 for each beam and the stress calculated by use of equation (5), which may be written in the form

$$f_v = \frac{Va}{Ka_v jd} = \frac{Vs \sin \alpha}{Ka_v\, jd} = c \cdot \frac{Vs}{a_v\, jd} \tag{10}$$

where the value of c is 0.707 when α is 45 deg., 0.724 when α is $32\frac{1}{2}$ deg., and 0.768 when α is 22 deg. A difficulty in applying this equation to beams with bent-up bars lies in the interpretation of the distance s, which is usually taken as the horizontal spacing of web members. Since the entire distance between the support and the inner load point is not fully covered by a series of parallel inclined bars, as assumed in the analysis, the portion of the distance that is reinforced by each inclined bar becomes problematic. Two assumptions may be made as to the distance s to be used: (1) The distance s may be taken as the horizontal spacing between layers of inclined bars, and a_v as the cross-sectional area of inclined bars in one layer. Thus in beam 384, having six inclined bars in three layers spaced eight inches apart, s is 8 in. and a_v is the area of two $\frac{5}{8}$-in. round

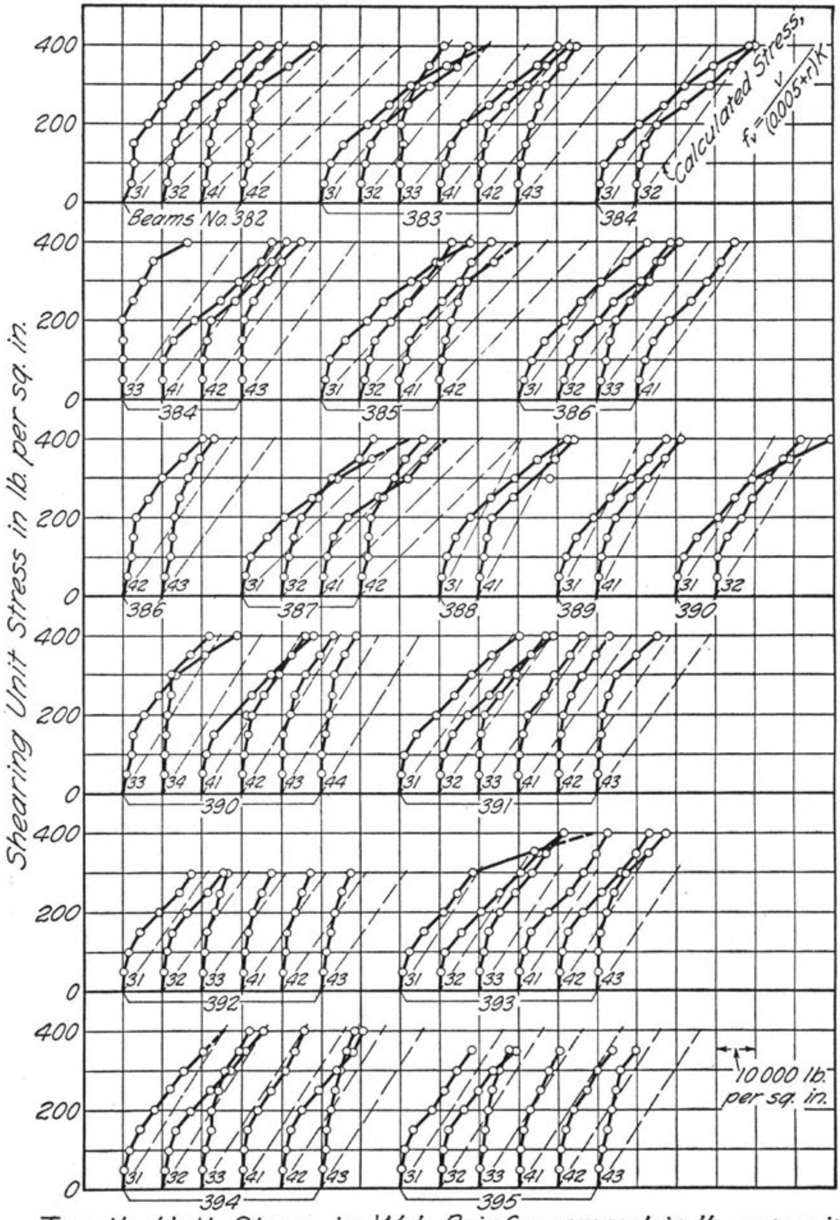

FIG. 21. LOAD-STRESS CURVES FOR INCLINED BARS, SERIES OF 1917

TABLE 7

MEASURED AND CALCULATED STRESSES IN WEB REINFORCEMENT

The numerals (1) and (2) indicate two assumptions used in the calculations. According to Assumption (1) the spacing s is the horizontal distance between layers of inclined bars; according to Assumption (2), s is taken as the horizontal distance between the extreme layers of inclined bars plus the depth, d, of the beam, divided by the number of layers of inclined bars.

Beam No.	Angle of Inclination, a, in deg.	Distance from Support to First Bend, in.	Spacing, s in.		Tensile Stress, f_v, lb. per sq. in.			
					Calculated,* by use of			Measured
			Assumption (1)	Assumption (2)	Eq. (10) Assumption (1)	Eq. (10) Assumption (2)	Eq. (11) Assumption (2)	On gage line No. 31
381	45	24	3	9.0	5 500	16 500	13 000	9 700
382	22	8	12	13.5	46 100	51 900	41 000	27 000
383	32½	8	12	13.0	43 400	47 000	32 400	38 000†
384	45	8	12	13.0	42 500	46 000	29 000	38 000†
385	32½	8	12	13.5	45 100	50 700	37 000	38 000†
386	32½	8	8	10.3	30 000	38 700	28 200	33 500
387	32½	8	16	15.5	60 100	58 200	40 700	38 000†
388	45	14	..	15.0	......	27 600	20 400	33 000
389	45	14	..	15.0	......	27 600	20 400	28 500
390	45	8	8	9.8	29 500	36 000	24 800	35 000
391	32½	12	8	10.3	30 000	38 800	28 300	31 000
392	32½	16	8	10.3	22 500	29 000	21 200	18 000
393	45	8	8	10.3	22 100	28 500	19 400	18 500
394	45	12	8	10.3	29 500	38 100	25 900	19 000
395	45	16	8	10.3	29 500	38 100	25 900	18 000
396	45	24	3	9.0	7 400	22 200	17 300	9 700
397	45	24	3	9.0	7 400	22 200	17 300	12 900
398	45	24	3	9.0	7 400	22 200	17 300	17 400
399	45	12	12	13.5	35 300	39 700	26 600	28 500
400	22	12	..	15.0	......	26 500	22 500	26 100

*Calculated stresses at a shearing stress of 300 lb. per sq. in. for beams 381, 392, and 393. 350 lb. per sq. in. for beam 400, and 400 lb. per sq. in. for the remaining beams.

†Measured stresses at or beyond the yield point of the steel.

bars. (2) The distance s may be taken as the sum of the horizontal distances included between the inclined bars plus a distance $d/2$ on each side of the group of inclined bars divided by the number of layers of bars. In beam 384, this would mean two spaces of eight inches included between the extreme inclined bars plus a distance d equal to 15 inches, making a total of 31 inches divided by 3, or 10.33 inches, while a_v is the area of two ⅝-in. bars as before. In either case the total distance effectively reinforced, which may be denoted by S, is equal to the sum of the individual distances s, and will be measured from the upper bend nearest the support toward the middle of the beam. Assumption (2) was made principally to account for the stresses in inclined bars placed in a single layer, a case to which the first assumption does not apply. Stresses computed by the use of both assumptions, together with corresponding stresses measured on gage line 31 at shearing stresses of 300 to 400 lb. per sq. in., are given in Table 7. It is seen that the stresses based on assumption (1) do not compare very consistently with the measured stress, the lack of agreement being greatest for the beams in which the inclined bars are in two closely spaced layers. It will be noted also that the measured stresses are frequently greater than the calculated ones, a relation that is contrary to the results of nearly all of the tests that have been made heretofore. A consideration of the stresses based on assumption (2) shows a somewhat better agreement between the measured and the calculated stresses; the stresses calculated for inclined bars in a single layer agree reasonably well with the measured stresses, and in most cases the ratio of measured to calculated stress varies between 0.6 and 0.9, with an average value of 0.74. This ratio agrees fairly well with the results of the series of 1911, as well as with previous tests.

Closer agreement between measured and calculated stresses might be expected from the use of equation (9), which is an empirical equation fitted to the results of other series of tests. Equation (9) was originally derived for tests with vertical stirrups, and with stirrups inclined at 45 deg.; it is evident that for stirrups inclined at other angles the equation should be modified by the introduction of the factor K as follows:

$$f_v = \frac{v}{(0.005 + r)K} \tag{11}$$

Stresses calculated by use of equation (11), based on assumption (2) regarding the spacing s, are included in Table 7. These values are nearer the measured stresses than those obtained by use of equation (10), though there are a few large variations for beams having inclined bars closely spaced or in one layer. The ratio of the measured stress to the stress calculated by use of equation (11) varies between extremes of 0.56 to 1.62, with an average value of 1.02. Stresses computed by use of equation (11) and assumption (2) are also plotted in Fig. 21, as shown by the broken lines. It is seen that there is fair agreement between measured and calculated stresses on gage lines 31, particularly for the higher loads. For other gage lines and low loads, the measured stresses are generally much less than the calculated ones. It may be concluded that by using assumption (2) to determine the spacing s, or range of effectiveness, of the web reinforcement, the stress f_v computed by use of equation (11) compares fairly well with the maximum stress measured in the inclined bars.

It is to be remembered that other variables were present that have not been included in any way in the computed stresses. For example, the distance from the support to the inclined bar undoubtedly affected the stress in the bar; as may be seen from Table 7 the highest ratios of measured to calculated stress were found for bars nearest the support and the smallest values for those farthest from the support. This is probably due to the fact that some of the stress in the inclined bar may be considered as a component of the longitudinal flexural stress, and this would also explain why the stress on gage line 31 was greater than on other gage lines farther from the support and in most cases nearer to the neutral surface.

Several causes of variation in stress will be considered in the following sections.

19. *Location of Inclined Bars.*—Six types of beams were made to study in particular the effect of placing the first bent-down bar at different distances from the support. Beams 386, 391, and 392 had six bars bent down at 32½ deg. in three layers spaced 8 in. apart horizontally, the first layer being bent down at 8, 12, and 16 inches from the support, respectively. Beams 393, 394, and 395 differed from the preceding three types in that the bars were bent down at 45 deg. instead of 32½ deg. Since the effect of the location of bends and the angle and spacing of bars are masked by the presence of flexural stresses, the variations in stress in web members will be studied in relation to the proximity of support, load points, and sections of contraflexure of the beam. In Fig. 22 diagrams of measured stresses in reinforcement have been plotted directly under the corresponding gage lines as shown on a sketch of each of the six types of beams under consideration. The diagrams show the variation in stress from top to bottom of each of the three layers of inclined bars, as well as the longitudinal stress at the support and at mid-span. The distance from support to first bend is 8 in. for the beams at the top of the figure, 12 in. for those at mid-height, and 16 in. for those at the bottom.

The measured stresses shown in Fig. 22 correspond to shearing unit stresses of 300 and 350 or 400 lb. per sq. in. As indicated in the figure the average yield point strength of the reinforcing steel was about 38 000 lb. per sq. in., so that bars in which the stress (calculated as proportional to the measured strain) exceeded this value were evidently stressed to the yield point. This explains the apparent variations in longitudinal stress near the support at the higher loads.

Figure 22 indicates that the stress in the inclined bars was greatest on the tension side of the beam and nearest the support, and least near the section of contraflexure. By comparing beams 386, 391, and 392, it is apparent that as the distance from the support to the first bend increased, the stress at the upper gage line on the first inclined bar decreased slightly; in general a similar variation was found on all

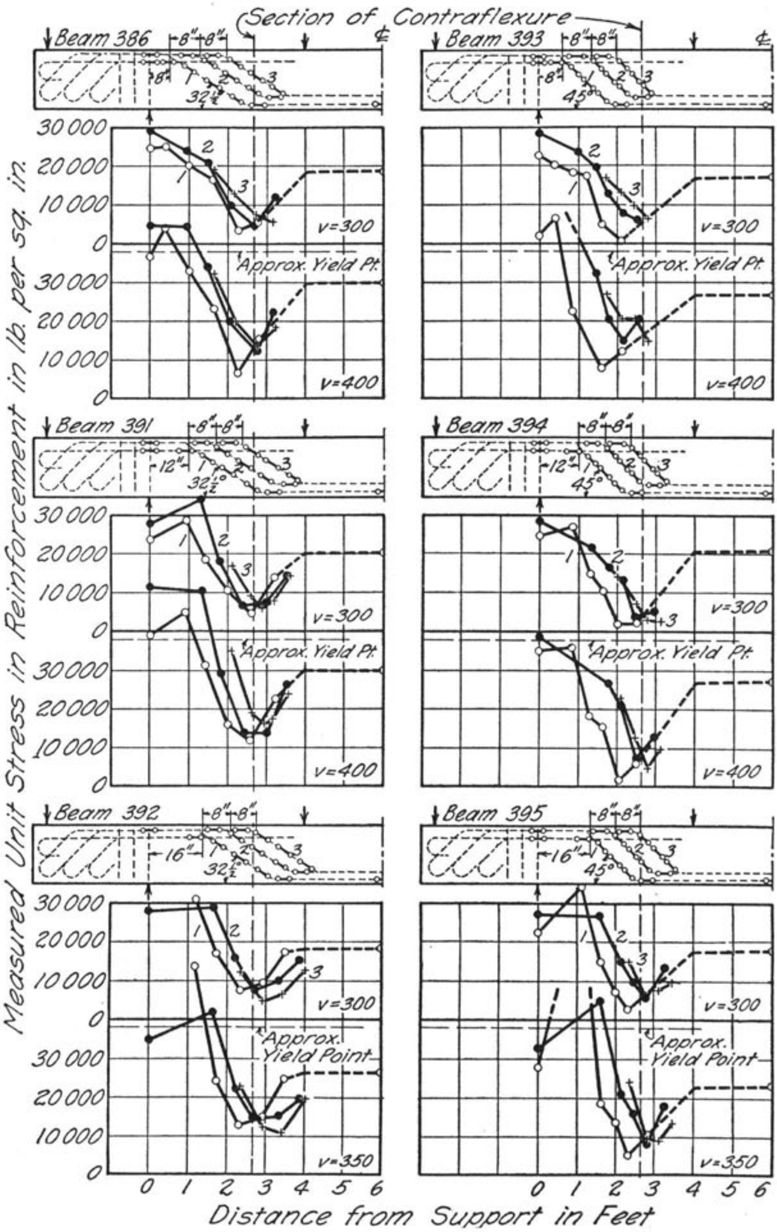

FIG. 22. MEASURED STRESSES IN REINFORCING BARS BENT DOWN AT DIFFERENT DISTANCES FROM SUPPORT

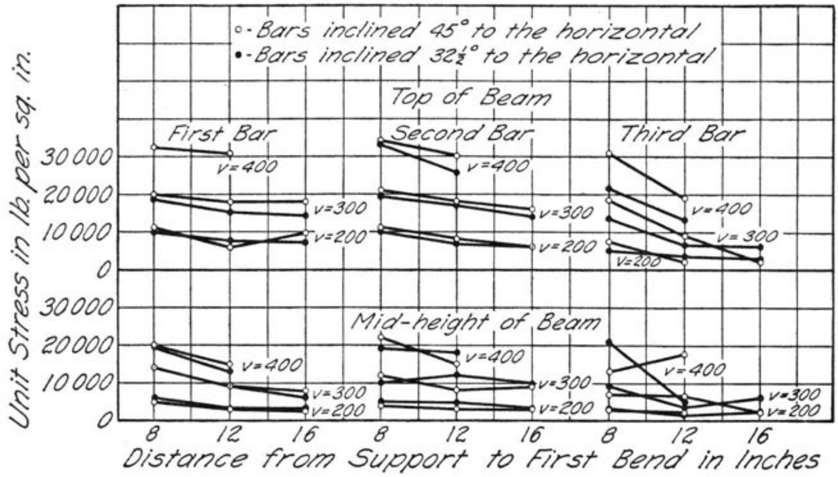

FIG. 23. MEASURED STRESSES IN INCLINED BARS AT UPPER AND MIDDLE GAGE LINES

gage lines on inclined bars. The stress in inclined bars near the section of contraflexure was comparatively low, being 6000 to 8000 lb. per sq. in. at a shearing stress of 300 lb. per sq. in., and 13 000 to 15 000 lb. per sq. in. at a shearing stress of 400 lb. per sq. in. Similar variations in stress were found in beams 393, 394, and 395.

A further comparison of the stresses in the inclined bars of the beams of Fig. 22 may be made by reference to Fig. 23, in which the measured stresses at the upper and middle gage lines on each of the three inclined bars are shown. It appears that the stress generally decreased as the distance from the support to the first bend increased; that the variation was quite regular for stresses in the first and second bars, but was somewhat erratic for the third bar from the support; and that the ratio of the stresses for angles of inclination of 32½ and 45 deg., respectively, was not far from the proportion indicated by equation (10).

The manner of failure of the beams of this group is also worthy of note. Referring to Section 16 and Figs. 10 to 19, it is found that in all cases a diagonal crack formed, beginning at or just inside the first bend, and extending toward the bottom of the beam at the support. In the beams having the first bend 8 or 12 in. from the support, failure was by tension in the longitudinal steel, followed by crushing of the concrete inside the bends; in the beams in which the distance was 16 in. the bent bars were obviously not in a position to act effectively as web reinforcement, although in three of the test beams the yield point of the longitudinal bars was reached before complete failure occurred along the diagonal crack. In the latter group of beams it is probable that additional web reinforcement in the region between the support and the first bend would have minimized the opening of diagonal cracks and thus would have given assurance against diagonal tension failure.

20. *Angle of Inclination of Web Bars.*—It is seen by reference to Table 5 that beams 382, 383, 384, and 385, as well as the six beams described in Section 19, were designed to give information on the effect of the angle of inclination of the bent-up bars upon their effectiveness as web reinforcement. The values of measured stresses in the inclined bars of these beams at shearing stresses of 300 and 400 lb. per sq. in. are shown in Figs. 22 and 24. As noted in Section 19, a part of the stress in the inclined bars is evidently due to flexure. The beam does not act as a simple truss such as is shown in Fig. 1, in which the web members are simple tension members, but due to the bending of the concrete and steel web as a whole, deformations are produced in the web members, in addition to the tensile stress indicated by equation (3). For a beam subjected to simple flexure alone, with no shearing stresses, it can be shown that the deformation along an inclined gage line is a simple function of the principal longitudinal deformations. Thus, for a short gage line inclined at an angle α to the horizontal, the unit deformation ϵ is equal to $\epsilon_1 \cos^2\alpha$ where ϵ_1 is the average longitudinal unit deformation at the middle of the gage line. Hence the ratio of the unit deformation on an inclined gage line to the average horizontal deformation at the same point is 0.50 for an inclination of 45 deg., 0.71 for 32½ deg., and 0.86 for 22 deg.

It is apparent that the unit deformation ϵ is a function of the bending moment and will be greatest near the support and inner load point, whereas that due to the stress f_v, as given by the equations of Section 3, is independent of the longitudinal stress. These deformations may evidently be coexistent rather than additive, so that the stress produced in a diagonal web bar will be that due to the greater of the two deformations rather than to the sum of them. On the upper gage lines on the inclined bars nearest the support it is evident that the calculated stress f_v was considerably augmented by the component of flexural stress.

The foregoing paragraph indicates that both flexural and shearing stresses should be considered in studying diagonal tension effects; in fact, a careful analysis of the diagonal tension stresses in a homogeneous beam would give many qualitative indications as to the action

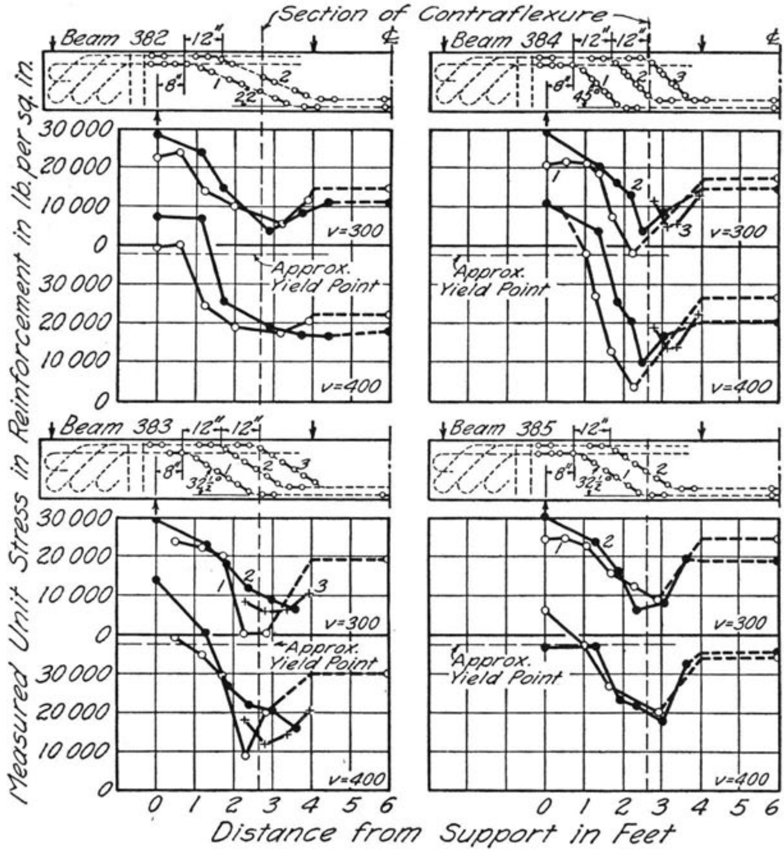

FIG. 24. MEASURED STRESSES IN BARS INCLINED AT DIFFERENT ANGLES

of a reinforced concrete beam, since previous to the formation of cracks the reinforced beam does not differ greatly in behavior from a plain beam, while the formation of cracks tends to restrict and concentrate the higher stresses in the reinforcement within those regions in which initial high concrete stresses occur.

While a correction for the effect of flexural stresses could be applied to the measured stresses obtained in these tests it is simpler for the purpose of comparing different arrangements of reinforcement to use only those stresses measured on gage lines near the neutral axis, or near the section of contraflexure, where the effect of flexure is negligible. Such a comparison of measured stresses near the sections of contraflexure of the four beams of Fig. 24 shows that for the angles of 22, 32½, and 45 deg., the respective measured stresses were about 8000, 10 000, and 11 000 lb. per sq. in. at a shearing stress of 300 lb. per sq. in. and 20 000, 21 000, and 20 000 lb. per sq. in. at a shearing stress of 400 lb. per sq. in. Whereas equation (10) indicates that the respective values should be in the proportions of 1.08, 1.03, and 1.00, these measured stresses are practically equal for the three angles. Even this, however, cannot be said to be a great difference from the theoretical proportions.

The greatest difference between measured and calculated stresses noted appears to be in the case of bars inclined 22 deg. to the horizontal. There are other indications that the use of this angle resulted in stresses disproportionately low as compared with those corresponding to other angles, as seen from a comparison of stresses measured at points away from the section of contraflexure. The additional effect of flexure on the stress in inclined bars should be to cause greater stress as the angle of inclination decreases; hence, due to this effect as well as that given by equation (10), the stress in bars inclined at 22 deg. at a given location in the beam should be larger than in bars inclined at larger angles. The measured stresses in beam 382, on the contrary, are generally less than the stresses at corresponding points in beams 383, 384, and 385, indicating that the effectiveness of the reinforcement inclined 22 deg. to the horizontal is less than that for the reinforcement inclined at the greater angles. Some of the apparent inconsistency noted in the action of these beams at a shearing stress of 400 lb. per sq. in. may be due to crushing of the concrete under bends and consequent local slip of bars, which was more pronounced in the beams with the larger angles of inclination of bars.

21. *Spacing of Web Bars.*—The effect of different spacings of inclined bars upon the web resistance may be studied by reference to the tests of beams 386, 383, and 387, in which the spacing between bends was made 8, 12, and 16 in., respectively. Values of the measured stresses in the reinforcement of these beams at a shearing stress of 400 lb. per sq. in. are shown in Fig. 25. The web reinforcement of the first two beams consisted of six bars bent down in three layers, while in the last one there were five bars bent down in two layers, two in the first layer and three in the second. The angle of inclination was $32\frac{1}{2}$ deg. to the horizontal for all.

As noted in Section 19, it is difficult to determine over what distance each inclined bar is effective; according to the assumptions used in Table 7 the stress in the bars spaced 8 in. apart should be considerably less than in those spaced 12 in. apart; this is not borne out by the curves of Fig. 25, though the stress near the section of contraflex-

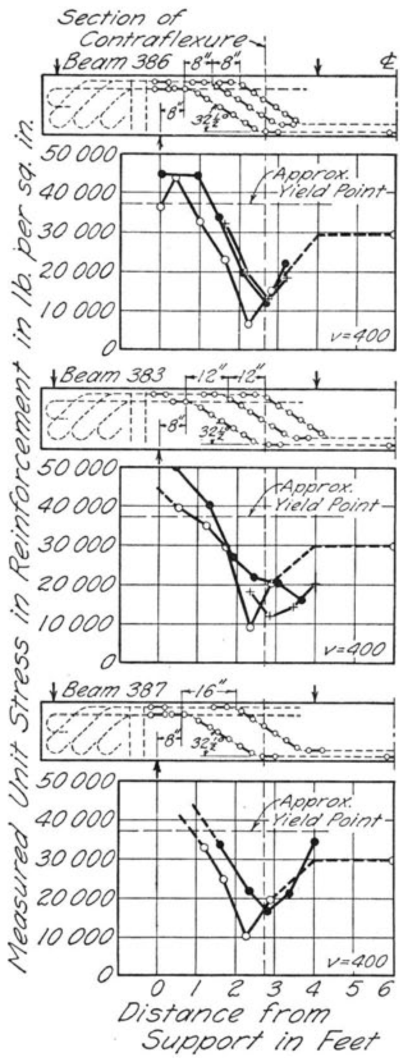

FIG. 25. MEASURED STRESSES IN INCLINED BARS AT DIFFERENT SPACINGS

ure in the two cases is in about the proper ratio. The stress in the bars spaced 16 in. apart seems to be unduly low in comparison with the other two spacings.

A second uncertainty in the comparisons is introduced by the fact that the different spacings used result in placing the bars in different parts of the beam. The section of contraflexure, for instance, intersects the inclined bars at different points in the three beams. A variation in web stress, which can not be accounted for as due to variation in the component of longitudinal flexural stress, may be seen by comparing inclined gage lines near mid-height of the beam, or near the neutral surface. The decrease in the web stress along the neutral line from support to section of contraflexure is probably caused by the gradual decrease in the number and size of diagonal cracks as the section of zero moment is approached. Since at any part of the beam the number of tension cracks, which may develop into diagonal cracks, is a function of the flexural stress present, it appears that the stress in inclined web members may be affected considerably, although indirectly, by the flexural stress.

It may be concluded that no great difference in effectiveness was found for the three spacings shown in Fig. 25, though the 16-in. spacing showed somewhat smaller stresses in comparison with those found with the 8-in. and 12-in. spacings than would be expected from theoretical considerations.

A similar comparison may be made between beam 393 and beam 384, Fig. 26, wherein spacings of 8 and 12 in. respectively, were used on bars inclined at 45 deg. to the horizontal. There appears to be little difference between the stresses in the first inclined bar from the support, while the stresses in the second and third bars are generally higher for the bars having the 8-in. spacing. Figure 26 also affords a comparison between the web stresses in beams 390 and 393, which were alike except that beam 390 contained an extra pair of inclined bars, bent down in the region of positive bending moment. The stresses in beam 390 are seen to be somewhat smaller than those in beam 393, though at the section of contraflexure they do not differ greatly. In the same figure, beam 399 may also be compared with beam 384, though the distance from support to first bend varies in the two beams. The stresses in the two do not differ greatly.

Three types of beams in the series had all of the inclined bars bent down in one layer. Figure 26 shows the measured stresses in the web reinforcement of these beams. Beams 388 and 389 were alike except for the presence of vertical stirrups in the latter as shown, while in beam 400 the bars were bent down at a rather small angle, 22 degrees. There were four inclined bars in each case. The stresses shown in Fig. 26 are not particularly noteworthy for beams 388 and 389; however, it may be noted that unusual slipping of the straight top bars occurred in beams 388.1 and 388.2, evidently due to high bond stresses produced by bending down so large a portion of the longitudinal reinforcement at one place. The point of bending down

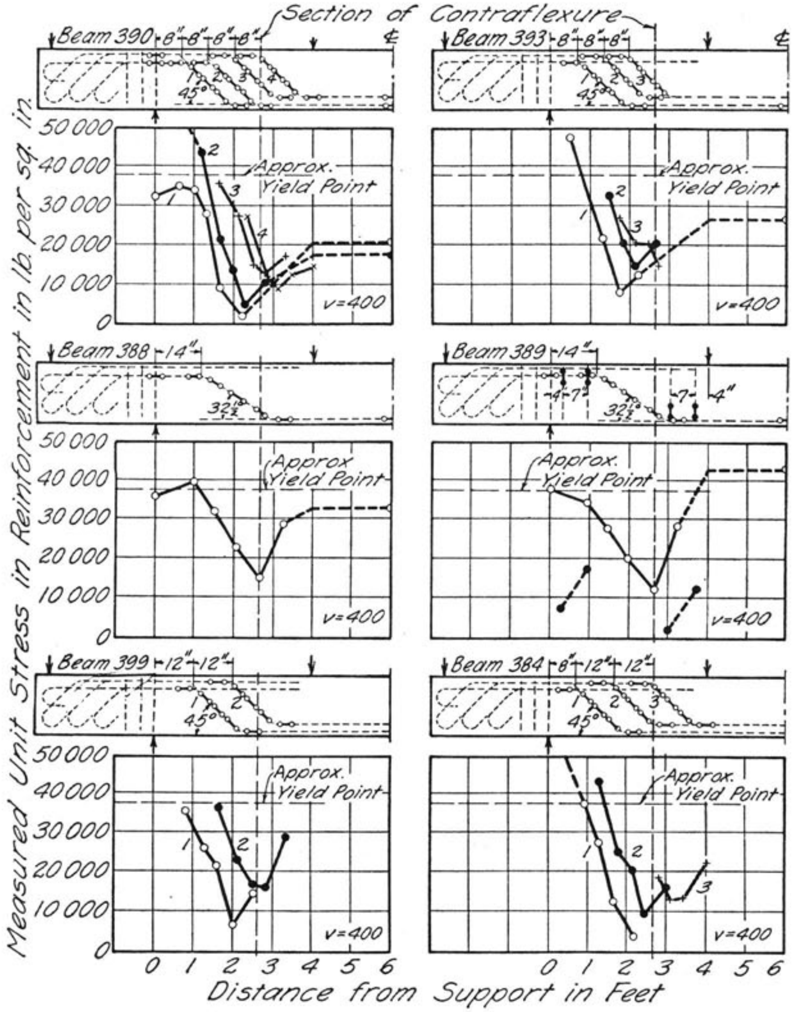

FIG. 26. MEASURED STRESSES IN REINFORCEMENT IN BEAMS OF VARIOUS TYPES

of the bars was also too far from the support to make the inclined bars very effective in preventing the opening of large diagonal cracks. Failure of beams 389.1, 400.1, and 400.2 occurred through the opening of diagonal cracks inclined at a small angle to the horizontal. In beams 400.1 and 400.2 high stresses were developed in the inclined bars and failure occurred through yielding of these bars at the comparatively low shearing stress of 364 lb. per sq. in. From the manner

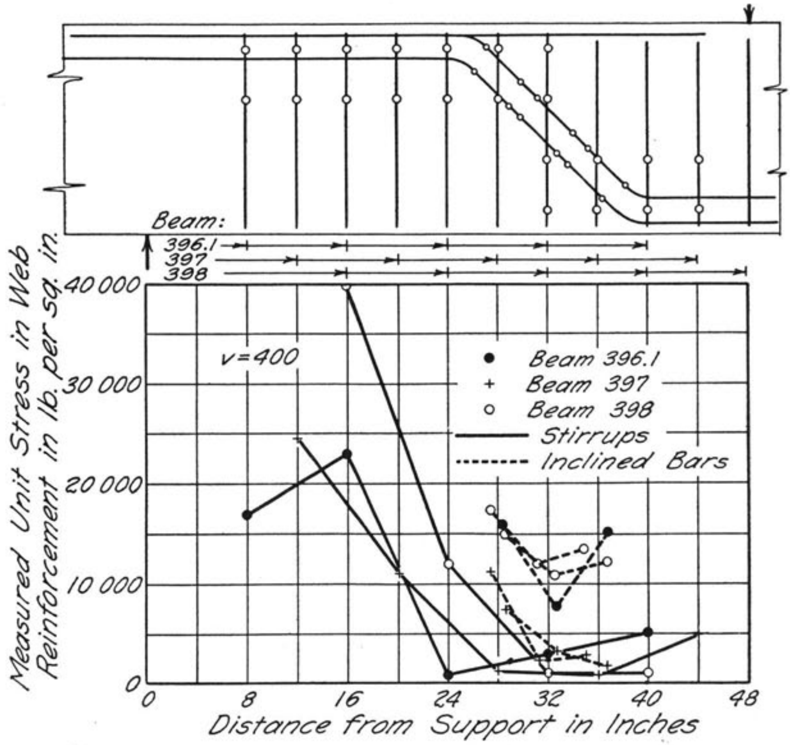

FIG. 27. MEASURED STRESSES IN WEB REINFORCEMENT WITH DIFFERENT LOCATIONS OF STIRRUPS

of failure of these beams the use of inclined bars in a single layer as a complete system of web reinforcement does not appear desirable.

22. *Location of Vertical Stirrups.*—Four beams were made in which ⅜-in. round double-U stirrups were used in addition to the inclined bars as web reinforcement. Beam 389 was made identical with beam 388 except that two stirrups were used near the support and two near the load point, as seen in Fig. 26. Beams 396, 397, and 398 were identical with beam 381, except that five stirrups, 8 inches apart, were used at each end of these three beams, the first stirrup being placed 8, 12, and 16 in., respectively, inside the support.

Figure 26 indicates that the stress in the inclined bars of beam 389 at a shearing stress of 400 lb. per sq. in. was slightly less than that in beam 388, but that the stress in the stirrups of beam 389 was low, varying from 7000 to 18 000 lb. per sq. in. near the support at a shearing stress of 400 lb. per sq. in. Figure 27 shows diagrammatically the maximum stresses measured in each of the stirrups of beams 396.1, 397, and 398 at a shearing stress of 400 lb. per sq. in. It appears that the stirrup receiving the highest stress was located 12 to 16 in. from the support or a distance approximately equal to the depth of the beam, and that the stress in stirrups more than 24 in. from the support was negligible. The stress in the first stirrup was 17 000 lb. per sq. in. when the distance was 12 in. and 40 000 lb. when the distance was 16 in. Obviously the last arrangement, that of beam 398, was much less effective in providing web resistance than were the other two.

The relation between the stirrup location that produced high stress and the location in which large diagonal cracks formed, as seen in Figs. 18 and 19, is quite apparent. Conversely, few cracks were noted near the section of contraflexure, when the stresses in stirrups were small.

For comparison with the measured stresses in the stirrups of beams 396, 397, and 398 at a shearing stress of 400 lb. per sq. in., the stresses have been calculated by use of equations (7) and (9). In all cases the spacing s is taken at 8 in., the area a_v is the cross-section of four $\frac{3}{8}$-in. round bars, or 0.441 sq. in., and b is 8 in., making r, or $\frac{a_v}{bs}$ equal to 0.0064. According to equation (7), f_v is equal to 62 500 lb. per sq. in., while according to equation (9), f_v is equal to 35 100 lb. It is seen that the latter stress compares fairly well with the highest stirrup stress measured in beam 398, which was 40 000 lb., or approximately the yield point stress for the stirrup. The stirrup stresses measured in beams 396 and 397 were considerably less than the calculated value.

The stresses in the inclined bars, which crossed the section of contraflexure at about mid-height of these beams, were also small, varying from 2000 to 17 000 lb. per sq. in. at a shearing stress of 400 lb. per sq. in., as shown in Fig. 27. The stresses in the inclined bars of beams 381.1 and 381.2, which were identical with the beams of Fig. 27 except that no stirrups were used, were also low, being 10 000 lb. per sq. in., or less, at a shearing stress of 260 lb. per sq. in.

23. *Stresses in Web Reinforcement at Section of Contraflexure.*—It has been remarked in the preceding sections that the measured stresses in inclined bars were generally lowest near the section of contraflexure. It is found by reference to Figs. 22, 24, 25, and 26 that at a shearing stress of 400 lb. per sq. in. the measured stresses at the section of contraflexure were generally less than one-half as great as

the maximum stresses, which were measured on gage line 31. In a homogeneous beam the diagonal tension at the section of contraflexure is inclined at 45 deg. to the horizontal and is equal in intensity to the shearing unit stress. It seems reasonable that in a reinforced concrete beam the tendency to form diagonal cracks is least at the section of contraflexure; furthermore this region is free from tension cracks which might develop into diagonal cracks. The absence of cracks is evidently in conformity with the low stresses observed in the web reinforcement, both stirrups and inclined bars.

24. *Crushing of Concrete at Bends.*—One of the most noticeable phenomena of failure observed in this series of tests was that of the crushing of the concrete inside the bends in reinforcing bars. As noted previously, all bends on the ⅝-in. round reinforcing bars were of 3-in. radius. According to the usual hoop-tension formula assuming the bearing pressure to be uniformly distributed inside the bend, and neglecting the effect of the stiffness of the bar, a stress in the bar equal to the average yield point strength of the steel, 38 000 lb. per sq. in., would produce a bearing pressure against the concrete of 6215 lb. per sq. in. While this bearing pressure is almost double the average compressive strength of the concrete as determined by tests of cylinders, it is not an excessive pressure for a small part of the beam well restrained by the surrounding concrete. However, in many of the test beams a bar was placed near the side of the beam to facilitate strain measurements, and the thin layer of concrete outside the bar was unable to provide the necessary restraint so that spalling and crushing of the concrete ensued. The consequence of even a small amount of yielding due to crushing was to cause progressive slipping of the bar due to the failure of bond at the bend and the high bond stresses at adjacent sections. Many of the views of the beams after failure show that as the bend straightened the bar stripped away from the concrete on the convex side of the bend.

It has been noted that diagonal cracks frequently intersected the reinforcement at bends. Obviously the high compression developed in the concrete inside the bend was accompanied by a tensile deformation or extension in a direction at right angles to the direction of compression. On the other hand, the presence of a diagonal tension crack running through the region of high bearing pressure apparently operated to destroy the condition of lateral restraint requisite to the development of high bearing strength, and thus furnished a starting point for crushing failure of the concrete. It is seen that the use of bent-up bars as web reinforcement may entail troublesome features in the design and fabrication of the bends, since the unfavorable combinations of tensile and compressive stresses existing at the bends are not conducive to the development of high bearing pressures in the concrete. Bends in highly stressed reinforcement should be designed with a radius sufficiently large to obviate the possibility of high bearing stresses.

V. SUMMARY

25. *Summary and Conclusions.*—This investigation embodies the tests of 59 restrained beams of reinforced concrete, of a size comparable to that of beams commonly used in buildings and other structures. The tests were arranged to produce conditions of restraint similar to those found in continuous beams. Considerable care was taken in securing even quality in the concrete used in the test beams, and the workmanship used in fabricating the reinforcement and in making the beams was uniformly good. In this respect the test beams are probably more free from defects which would seriously affect the strength than are beams commonly encountered in construction work.

The experiments were made from 1911 to 1917, a period that marked the development of the strain gage into an almost indispensable instrument of structural research. In only one of the earlier published investigations of web stresses were strain gage measurements secured. The measured strains obtained in this investigation have been invaluable in showing definitely what was the primary, and what the secondary, cause of failure, as well as in indicating the potential web resistance of beams that failed in different ways. The following are the principal conclusions to be drawn from the investigation:

(1) The measured stresses in web members were quite variable; the magnitude of the stress depended to a considerable extent on where the web member was intersected by cracks in the concrete. Generally, the highest measured stresses agreed roughly with a single empirical relation as given by equation (11), page 57. The stresses measured on a majority of the gage lines, and in fact all stresses measured at the lower loads, were considerably less than those given by equation (11).

(2) In beams with vertical stirrups the greatest stirrup stresses were observed at a distance from the center of the support about equal to the depth of the beam. This location of the greatest stress may be due in part to the fact that, as these beams were designed, the stress in the longitudinal reinforcement was usually considerably greater at the support than at mid-span.

Views of the beams after test show cracks radiating from the support in a fan-like pattern; the cracks that were inclined at angles greater than 45 deg. to the horizontal caused concentrations of stress principally in the longitudinal reinforcement; the cracks inclined at 45 deg. or less at approximately a distance *d* from the support induced high stresses in the web reinforcement.

(3) In the beams reinforced with bent-up bars the region of maximum web stresses is not very clearly defined, but generally the highest stresses in inclined bars were found when the upper bend of the bar was less than a distance *d* from the support. The greatest stress in an inclined bar was almost invariably found in the part nearest the tension face of the beam. In the inclined bars there are many cases in which at the highest loads a crushing of the concrete and local slipping of the bar at the upper bends allowed longitudinal stress to be transmitted around the bend into the inclined portion of the bar.

(4) The stresses in both stirrups and inclined bars were low near the section of contraflexure as compared with the stresses measured elsewhere. There were few tension cracks in this region to provide a starting point for diagonal cracks and this absence of cracks is in accord with the low stresses observed. In only a few cases large diagonal cracks formed across the section of contraflexure and produced complete failure. In these cases the web reinforcement was not distributed effectively to resist the diagonal tension.

(5) It is a function of web reinforcement to keep the diagonal cracks well distributed and small in size, both from considerations of appearance and permanence and to prevent large concentrations of stress which might lead to premature failure. The 1917 tests showed that, when properly arranged, bent-up bars furnished effective web reinforcement. Bent-up bars in a single layer evidently did not furnish complete web reinforcement, as indicated by the manner of failure, by the distribution of cracks, and by the stresses developed. Of the two types of beams in which inclined bars in one layer formed the only web reinforcement, one failed by diagonal tension and the other by combinations of bond, tension, diagonal tension, and crushing.

(6) In the beams reinforced by bent-up bars, the distance of the first bend from the support had an important effect. When the bars were inclined at 32½ deg. to the horizontal, diagonal

tension failure occurred when the distance was 16 in., and a secondary diagonal failure occurred when the distance was 12 in., or 0.8 d. With the angle of inclination equal to 45 deg. a secondary diagonal tension failure occurred when the distance was 16 in., and in one case when the distance was 8 in. Since large diagonal cracks frequently intersected the inclined bar at the upper bend where the inclination was small, it appears desirable that the distance from the support to the first bend should not exceed one-half of the depth of the beam.

(7) According to analysis, as represented by equation (10), the stresses in bars of equal size, having equal horizontal spacing, and inclined at angles of 45, 32½, and 22 deg., respectively, should be slightly greater for the smaller angles. The measured stresses compared fairly well with the theoretical ratios for inclinations of 45 and 32½ deg., but for the inclination of 22 deg. the measured stresses were generally somewhat less than would be indicated by equation (10), indicating less effectiveness for this angle of inclination.

(8) When inclined bars are used to reinforce a given portion of a beam their spacing should naturally be such that a diagonal crack could not form without intersecting one or more of the inclined bars. The horizontal spacings used in the series of 1917 exceeded 12 in., or 0.8 d, in only one case. The 16-in. spacing used in this one type of beam seemed fairly satisfactory, although the stresses in the steel were lower than they were expected to be. For the purpose of governing the distribution of cracks it would seem advisable to limit the horizontal spacing of inclined bars to three-fourths of the depth of the beam.

(9) When inclined bars are placed in one layer to act as part of a web reinforcing system, the distance along the beam that may be considered to be effectively reinforced is not indicated by theoretical considerations; from a study of the stresses measured in all of the beams it seems reasonable to consider the bars effective over a distance not exceeding the depth d of the beam. This distance would ordinarily be measured from the bend on the tension side of the beam toward the section of contraflexure.

(10) The 1911 tests demonstrated that when reinforcing bars were rigidly supported during pouring (particularly the bars near the top of the beam) the concrete settled away from beneath the bars during the setting period, causing cavities or fissures which destroyed a large part of the effective bond area. These fissures

were doubtless responsible for the early slip of reinforcing bars near the support, prevalent in this series of tests. The slip was evidently due to a progressive bond failure, starting on the straight bars near the support or else where high bond stresses were caused by bending down anchored bars, and extending toward the inner and outer load points. This initial bond failure induced secondary tension and diagonal tension failures, due to the unusually large deformation occurring in the vicinity of the support due to the shifting of stress from the unanchored to the anchored bars.

(11) No slipping of vertical stirrups was observed in any of the tests and no slip of consequence of web or longitudinal steel was noted in the 1917 tests, except that following the crushing of the concrete inside bends at comparatively high loads.

(12) Anchorage of top reinforcing bars by bending them down and providing large hooks in the compression side of the beams generally prevented final bond failure. End anchorage did not prevent local slipping of bars, but the use of small bars as in the 1917 tests seemed to be quite effective in this respect. Excessive bond stresses must be avoided if the full strength of the web reinforcement is to be developed.

(13) The crushing of the concrete due to high bearing pressures inside of bends in the reinforcement was a noticeable feature of the 1917 tests. The radius of curvature of these bends, which was equal to 4.8 times the diameter of the bar, was evidently too small to develop the yield point strength of the bar without crushing the concrete. To this statement may be added the facts that some bends were too close to the vertical surface of the beam, and that lateral restraint of the concrete in bearing was destroyed by diagonal cracks intersecting the bend. The advantage in the use of bends of large radius in reinforcing bars is obvious.

(14) The values of the maximum shearing stress developed in restrained beams without web reinforcement varied from 142 to 260 lb. per sq. in. in the two series of tests, though in the case of the lower values, failure was by tension and bond. Values of the maximum shearing stress developed in beams with web reinforcement are not particularly significant, due to the diversity of causes of initial failure. The information obtained from these tests does not indicate an upper limit on the shearing stresses that may be allowed in beams in which the web reinforcement satisfies

the usual design formulas, provided failure due to other causes is precluded.

(15) The prevalence of local bond failures at low loads and the resulting formation of cracks, and of the crushing and splitting produced by bends of small radius, indicates that if high allowable shearing stresses are to be utilized in design, particular attention must be paid to matters of longitudinal tension, bond, and bearing stresses. Emphasis is lent to this statement by the fact that test specimens in this and other investigations which were designed to fail by diagonal tension very frequently failed from other causes.

(16) In beams of the proportions tested, the longitudinal deformations and accompanying tension cracks have a decided bearing upon the development of diagonal tension cracks and the resulting stress in web members. It is evident that if tension cracks can be inhibited or minimized greater effectiveness of the web reinforcement will be secured.

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